Chapter 6: Economic Inequality

We are interested in inequality mainly for two reasons: First, there are philosophical and ethical grounds for aversion to inequality per se. Second, even if we are not interested in inequality at an intrinsic level, we may care about inequality at a functional level (i.e. inequality may be important not for its own sake, but because it has an impact other economic features that you care about, such as growth).

**Question:** How should we think about inequality at a conceptual level?

We will focus on inequality of income or wealth, although, as we mentioned before, inequality is intimately linked to concepts such as lifetimes, personal capabilities and political freedoms.

We will focus on inequality of income or wealth, not because they represent all differences, but because they represent an important component of those differences.

**Short-term vs. long-term considerations:** Inequality in current income may be harmless, both from an ethical point of view and from the point of view of their effects on the economy. For example, suppose in society A that some people earn $2000/month while others earn $3000/month and there is perfect immobility. In society B, some earn $1000/month while others earn $4000/month but people exchange jobs every month. At any one point in time, society A shows up as more equal, but in society B average annual earnings are the same for everyone. (Data problems.)

**Personal vs. functional distribution of income:** How much people earn vs. how they earn it (rent, interest, wage). Related to the ownership pattern of productive factors (land, capital, labor). Looking at income inequality may mask serious differences, such as the well-being of an unemployed person who receives his income from charity and a person who earns himself the same amount of income. Also, judge whether two middle-class families with the same amount of income should be deemed equal (one receives rent income whereas the other receives wage income).

**What to use as an indicator of resources? Income vs. wealth vs. expenditures:**

If income is chosen as the indicator, it is usually the disposable income that is used. Disposable income is the income that households have available for spending and saving after income taxes have been accounted for.

Wealth can be defined as net worth, which is the sum of assets (including financial and non-financial assets minus the liabilities of the economic unit. It is a summary measure of all accumulated resources at a given time.

Some researchers think that income is an indirect measure of the standard of living and thus prefer to use expenditures as the indicator. Expenditures include current expenditures (purchases of goods and services, such as food, rent, utilities, entertainment and personal care) and capital expenditures (such as the repayment of the principal on own home, the purchase of investment properties, home improvements and life insurance). Both income and expenditures are flow variables, whereas wealth is a stock variable.

**Question:** How should we measure inequality?
Measuring Economic Inequality

Suppose that there are only two people. If they share a cake 50-50 then the distribution is equal, otherwise it is not. Furthermore, we can easily see that a 40-60 split is more equal than a 30-70 split.

However if there are more than two people, then making a judgment becomes hard. How do you compare a 20-30-50 division with a 22-22-56 division?

We need a measure that will help us rank different distributions. Let \((y_1, y_2, \ldots, y_n)\) be an income distribution and \(I = I(y_1, y_2, \ldots, y_n)\) be an inequality measure, defined as a function of the income distribution.

Question: What properties should a desirable inequality measure satisfy? Should we have weak or strict criteria?

Four criteria for inequality measurement:

1. Anonymity principle (sometimes called the symmetry principle), which says that it does not matter who receives the income. Permutations of incomes among people should not matter for inequality judgments. This means that we can always arrange the income distribution of \(n\) people so that \(y_1 \leq y_2 \leq \ldots \leq y_n\), where \(y\) is income. If the prime minister trades his income with one of yours, income inequality in Turkey should not change.

2. Population principle, which says that cloning the entire population and their incomes should not change inequality. According to this principle, population size does not matter. All that matters are the proportions of the population that earn different levels of income. For example, suppose that we arrange and categorize the incomes so that we can determine the percentage of the population earning income in each category.

<table>
<thead>
<tr>
<th>Income range ($)</th>
<th>Percentage of the population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-200</td>
<td>8</td>
</tr>
<tr>
<td>201-400</td>
<td>30</td>
</tr>
<tr>
<td>401-600</td>
<td>33</td>
</tr>
<tr>
<td>601-800</td>
<td>25</td>
</tr>
<tr>
<td>801+</td>
<td>4</td>
</tr>
</tbody>
</table>

Here, for inequality measurement purposes, we do not know (and do not care) how many people live in the country or who they are (the anonymity principle). All that we care about is the percentage of the population earning different levels of income.

Formally, we can state this principle as

\[
I(y_1, y_2, \ldots, y_n) = I(y_1, y_2, \ldots, y_n; y_1, y_2, \ldots, y_n)
\]
3. Relative income principle, which says that only relative incomes, and not the absolute levels of these incomes matter. For example, an income distribution over two people of (500, 1000) has the same inequality as (2000, 4000), and regardless of the currency the incomes are denominated in.

This principle is actually not very easy to buy. Absolute incomes are important determinants of overall economic development. Later in the course, we will make up by focusing explicitly on absolute incomes!! 😊

With the relative income principle added, we do not need absolute income information. We can now define income categories in terms of quintiles (or deciles or percentiles, depending on data availability).

<table>
<thead>
<tr>
<th>Income Quintile</th>
<th>Percentage of total income earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poorest quintile</td>
<td>9</td>
</tr>
<tr>
<td>Second quintile</td>
<td>15</td>
</tr>
<tr>
<td>Third quintile</td>
<td>20</td>
</tr>
<tr>
<td>Fourth quintile</td>
<td>25</td>
</tr>
<tr>
<td>Richest quintile</td>
<td>31</td>
</tr>
</tbody>
</table>

Formally, we can state this principle as

\[ I(y_1, y_2, \ldots, y_n) = I(\lambda y_1, \lambda y_2, \ldots, \lambda y_n), \text{ for every positive number } \lambda. \]

4. The Dalton principle (also known as “The Pigou-Dalton transfer principle”):

Let \((y_1, y_2, \ldots, y_n)\) be an income distribution and consider two incomes \(y_i\) and \(y_j\), \(y_i \leq y_j\). A transfer of income from the “not richer” individual to the “not poorer” individual will be called a regressive transfer. (The opposite of a regressive transfer is a progressive transfer.)

The Dalton principle states that if one income distribution can be generated from another via a sequence of regressive transfers, then the original distribution is more equal than the other.

Formally, we say that I satisfies the Dalton principle if for every income distribution,

\[ I(y_1, \ldots, y_i, \ldots, y_j, \ldots, y_n) < I(y_1, \ldots, y_i - \delta, \ldots, y_j + \delta, \ldots, y_n), \text{ for every positive number } \delta. \]

The Lorenz Curve:

First we rank the economic units according to income (or another measure of resources) from the smallest to the largest. Then we plot the cumulative shares of units versus the cumulative share of these units in total income. In particular, the horizontal axis shows the cumulative percentages of the population arranged in increasing order of income. On the vertical axis, we show the percentage of total income received by a fraction of the population. The convex shaped curve thus drawn is called the Lorenz curve. For example if the (20%, 6%) point is on the Lorenz curve, then this means the poorest 20% of the population receives 6% of total income. This also means that the richest 80% of the population receives 94% of total income.
The greater the level of inequality, the farther the Lorenz curve will be from the 45 degree line. In the figure, L2 lies to the right of L1 everywhere, so we would expect an inequality index to indicate greater inequality in the L2 case. Another way to see this is that, the poorest x% of the population will always have an equal or greater share of income under L1 than under L2, regardless of what x is. This is called the Lorenz criterion (or Lorenz dominance criterion).

Formally, an inequality measure $I$ is consistent with the Lorenz criterion, if for every pair of income distributions $(y_1, y_2, \ldots, y_n)$ and $(z_1, z_2, \ldots, z_m)$,

$I(y_1, y_2, \ldots, y_n) \geq I(z_1, z_2, \ldots, z_m)$, whenever the Lorenz curve of $(y_1, y_2, \ldots, y_n)$ lies to the right of $(z_1, z_2, \ldots, z_m)$ everywhere.

It turns out that an inequality measure is consistent with the Lorenz criterion if and only if it satisfies all of the four principles that we stated above: the anonymity, population, relative income and Dalton principles. Therefore, the Lorenz criterion captures our four criteria in one statement (and in graphical form). If the Lorenz curves of two distributions do not cross, then all inequality measures that are consistent with the Lorenz criterion yield the same ranking of these distributions. If two Lorenz curves cross, then inequality measures that are consistent with the Lorenz criterion may yield different rankings.

**Exercise:** Show on a graph that if an income distribution satisfies the Dalton principle (along with the other three principles), it is also consistent with the Lorenz criterion. (Hint: Imagine a regressive income transfer and show how it affects the Lorenz curve.)

**Question:** What happens if Lorenz curves cross?

**Answer:** In this case, the Lorenz criterion does not apply. When there is a Lorenz curve crossing, we cannot go from one distribution to another via a sequence of regressive transfers; we must have at least one progressive transfer.

**Example:** Suppose that a society consists of four individuals with incomes (75, 125, 200, 600). Consider a second income distribution (25, 175, 400, 400). How can we travel from the first to the second?
You will notice that the answer will include at least one regressive and one progressive transfer. In other words, we will not be able to compare the two distributions by using the four principles. What we need here is another principle that allows us to weigh the costs of a regressive transfer with the benefits of a progressive transfer. This requires a value judgment which makes it impossible to quantify the trade-offs in a way that is approved by everyone!

Complete measures of inequality:
Although Lorenz curve is a nice tool to visually examine the degree of inequality, often there is a need to summarize the information in a number. A number is more concrete and quantifiable than a picture. Moreover, this number can be used to make a judgment when Lorenz curves cross.

Let us suppose that there are n individuals and m distinct incomes. In each income class j, the number of individuals earning that income is $n_j$. Therefore, $n = \sum_{j=1}^{m} n_j$.

Define the overall mean income as $\mu = \frac{1}{n} \sum_{j=1}^{m} n_j y_j$.

Some complete measures of inequality that are often used are:

**Range:** $R \equiv \frac{1}{\mu} (y_m - y_1)$

**The Kuznets ratios:** The ratio of the shares of income of the richest x% to the poorest y%.
For example, the ratio of the income of the richest 10% to the income of the poorest 40% can tell us something about distribution when more detailed information is not available.

The range and the Kuznets ratios are crude but nevertheless useful indicators of inequality when detailed data are not available.

**The mean absolute deviation:** Express average deviation from mean income as a fraction of total income.

$$M \equiv \frac{1}{n\mu} \sum_{j=1}^{m} n_j |y_j - \mu|$$

Although it takes into account the entire income distribution (unlike the previous two), the mean absolute deviation is actually not a very good measure of inequality. It has one serious drawback; moreover it does not have a compensatory feature. The drawback is that it often does not satisfy the Dalton principle. To see this, take two incomes, both either above the mean or below the mean. A regressive transfer between the two (so that both incomes remain either above or below the mean, as before) will leave the mean absolute deviation unchanged.

**The coefficient of variation:** It is the ratio of the standard deviation to the mean.

$$C = \frac{1}{\mu} \sqrt{\frac{1}{n} \sum_{j=1}^{m} n_j (y_j - \mu)^2}$$
This measure satisfies all four properties, so it is Lorenz-consistent.

**The Gini coefficient**: Very widely used in empirical work. It takes the absolute differences between all income pairs and sums them up. The sum of differences is divided by \( n^2 \) (since there are \( n^2 \) such pairs), by the mean (so that the coefficient is sensitive to only relative changes in incomes and not to absolute changes), and by two (since each income pair appears twice).

\[
G = \frac{1}{2n^2 \mu} \sum_{j=1}^{m} \sum_{k=1}^{m} n_j n_k |y_j - y_k|
\]

This measure satisfies all four properties, so it is Lorenz-consistent. Check and make sure!

We also know that the Gini coefficient is the ratio of the area between the Lorenz curve and the 45-degree line to the area of the triangle below the 45-degree line.

The last two measures of inequality, namely the coefficient of variation and the Gini coefficient are Lorenz-consistent. Therefore, both are satisfactory according to our criteria. Which one should we use?

The crucial point is that both measures will give the same ranking as long as Lorenz curves can be ranked, i.e. they do not cross. When the Lorenz curves cross, it is possible for the two measures to give different rankings. In such cases, we should rely on not one but a set of measures, and probably study the Lorenz curves as well.

*(Note: If there is no Lorenz crossing, then Gini and CV (and other measures that are Lorenz-consistent) agree. If Gini and CV do not agree, then there is Lorenz curve crossing. If Gini and CV agree, there may or may not be Lorenz curve crossing.)*

<table>
<thead>
<tr>
<th>Country /date</th>
<th>Gini</th>
<th>Coefficient of variation</th>
<th>Income share of the richest 5% (%)</th>
<th>Income share of the poorest 40% (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Puerto Rico</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>0.415</td>
<td>1.152</td>
<td>23.4</td>
<td>15.5</td>
</tr>
<tr>
<td>1963</td>
<td>0.449</td>
<td>1.035</td>
<td>22.0</td>
<td>13.7</td>
</tr>
<tr>
<td>Argentina</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>0.412</td>
<td>1.612</td>
<td>27.2</td>
<td>18.1</td>
</tr>
<tr>
<td>1959</td>
<td>0.463</td>
<td>1.887</td>
<td>31.8</td>
<td>16.4</td>
</tr>
<tr>
<td>1961</td>
<td>0.434</td>
<td>1.605</td>
<td>29.4</td>
<td>17.4</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1950</td>
<td>0.526</td>
<td>2.500</td>
<td>40.0</td>
<td>14.3</td>
</tr>
<tr>
<td>1957</td>
<td>0.551</td>
<td>1.652</td>
<td>37.0</td>
<td>11.3</td>
</tr>
<tr>
<td>1963</td>
<td>0.543</td>
<td>1.380</td>
<td>28.8</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Source: Fields (1980).

In the table above, we can see that in Puerto Rico, both the poorest and the richest lost income share. This is a clear sign of Lorenz crossing. Gini and CV disagree in this case, although a Lorenz curve crossing does not necessarily mean that Gini and CV disagree. In Argentina, in 1953-61, Gini and CV do not agree, therefore there must have been a Lorenz crossing, although the population shares do not show this to us. In Mexico, in 1957-63, Gini and CV agree, but the changes in population shares indicate that there has been a Lorenz curve crossing.