Chapter 4: The New Growth Theories

References: Debraj Ray, Development Economics; Barro and Sala-i Martin, Economic Growth; David Weil, Economic Growth.

Some concerns that we have so far about growth theory are:

1. The Solow model can explain only part of the differences in per capita income. The remaining differences suggest that there are non-diminishing returns to physical capital, which is hard to accept. We know that the share of physical capital in total product is not very high.
2. The theory assumes that there are differences in key parameters without explaining why they are different.
3. Technical progress, which determines long-run growth rates, is actually made by conscious actions of people, and therefore should not be regarded as exogenous. Moreover, it may not be reasonable to assume that technology flows freely among countries.

Human Capital and Growth

Assume for simplicity that population is constant and that there is no depreciation. Augment the Solow model by introducing human capital.

\[ y = k^\alpha h^{1-\alpha} \]

Human capital, in contrast to labor, is deliberately accumulated and is not the simple outcome of population growth.

Allow individuals to save in two forms: physical capital and human capital.

\[
\begin{align*}
    k_{t+1} - k_t &= sy_t \\
    h_{t+1} - h_t &= qy_t
\end{align*}
\]

Define \( r \equiv h/k \).

Let’s figure out the growth rate in k, h and y.

\[
\begin{align*}
    \frac{k_{t+1} - k_t}{k_t} &= s \frac{k_t^{\alpha} h_t^{1-\alpha}}{k_t} = sr^{1-\alpha} \\
    \frac{h_{t+1} - h_t}{h_t} &= q \frac{k_t^{\alpha} h_t^{1-\alpha}}{h_t} = qr^{-\alpha}
\end{align*}
\]

How will \( r \equiv h/k \) evolve?

\[
\frac{r_{t+1}}{r_t} = \frac{h_{t+1}/h_t}{k_{t+1}/k_t} = \frac{qr^{-\alpha} + 1}{sr^{1-\alpha} + 1}
\]

Dividing both the numerator and the denominator by \( r_t^{1-\alpha} \), we get

\[
\frac{r_{t+1}}{r_t} = \frac{q/r_t + r_t^{\alpha-1}}{s + r_t^{\alpha-1}}.
\]
We can see that if \( r_t > \frac{q}{s} \), then \( 1 > \frac{r_{t+1}}{r_t} > \frac{q/r_t}{s} \), in other words, \( r_t > r_{t+1} > \frac{q}{s} \), \( r \) is decreasing.

On the other hand, if \( r_t < \frac{q}{s} \), then \( 1 < \frac{r_{t+1}}{r_t} < \frac{q/r_t}{s} \), in other words, \( r_t < r_{t+1} < \frac{q}{s} \), \( r \) is increasing.

Once \( r_t = \frac{q}{s} \), it stays there.

In fact, this makes perfect sense. At steady-state, both \( h \) and \( k \) should be growing at the same rate. Therefore we can write \( sr^{1-a} = qr^{-a} \), which means that \( r = \frac{q}{s} \).

Looking at the above equation, we can say that the larger is the ratio of saving in human capital to saving in physical capital, the larger is the long-run ratio of human to physical capital. The steady-state growth rate of variables \( h, k \) and \( y \) is:

\[
sr^{1-a} = s(q/s)^{1-a} = s^a q^{1-a}
\]

This model has several implications:

1. It is perfectly possible that there are diminishing returns to capital, yet no convergence in per capita income. Even when countries have similar saving and technological parameters, we should not expect to see any tendency for their per capita incomes to converge. They only grow at the same rate in the long-run due to having the same rate of technical progress.

Remember that the empirical testing of the Solow model showed that the world behaved as if there are constant returns to capital, but we are reluctant to accept this argument. The reconciliation to this paradox is the following: There can be diminishing returns to physical capital alone but constant returns to physical and human capital combined. (To see this, increase the amounts of \( k \) and \( h \) by a factor \( \lambda > 0 \) in the production function.)

2. With CRS, the \( s \) and \( q \) parameters have growth-rate effects, and not just level effects. In other words, the long-run growth rate is determined from within the model, by the parameters of the model. This is why we call such models endogenous growth theories. In this sense, the Harrod-Domar model was the first example of endogenous growth theory! However, unlike the Harrod-Domar model, the present theory has diminishing returns to each input separately.

3. Note that the growth effects in item 2 are related to the constancy of returns in physical and human capital combined. If we introduced a third factor that grows exogenously, such as labor, the resulting model would look like the Solow model. The reason is that in that case physical and human capital combined would exhibit diminishing returns.

4. Another implication (which can be tested) is the following: In the long-run, the ratio of \( h \) to \( k \) is known \( (q/s) \). This means that if a country has a low level of \( k \) relative to its \( h \), it will tend to grow faster in per capita terms, ceteris paribus.

This leads to two predictions:
a) Conditional convergence after controlling for human capital: Conditioning on the level of human capital, poor countries will tend to grow faster.

b) Conditional divergence after controlling for the initial level of per capita income: Conditioning on the level of per capita income, countries with more human capital will tend to grow faster.

When these two effects are combined, the model gives us neutrality of growth rates with respect to per capita income.

The empirical testing of the above is done as follows (Barro, 1991, QJE):
Regress the average growth rate in per capita real GDP over the period 1965-85 on per capita GDP in 1960 and school enrollment rates (among other variables). (Testing for conditional convergence here.) The results indicate that conditioning for human capital, the coefficient on initial per capita GDP is negative and significant, while conditioning on initial per capita GDP the coefficient on human capital is positive and significant.

This finding means that a plot of average growth rates against initial per capita income essentially picks up two effects. First, a high initial income slows down the growth rate, and second, higher level of human capital speeds up the growth rate. When combined, the two tend to cancel each other out.

See the graphs and regression results in the following pages. In the regression output, eight different specifications are shown. You will see that the partial effect of initial per capita GDP is negative (for example, the coefficient of the GDP60 variable in the first specification is -0.0075), and the partial effect of human capital is positive (for example, the coefficient of the PRIM60 variable in the first specification is +0.025).

The following is from Barro (QJE, 1991), “Economic growth in a cross-section of countries”:

“In neoclassical growth models, such as Solow (1956), a country's per capita growth rate tends to be inversely related to its starting level of income per person. In particular, if countries are similar with respect to structural parameters for preferences and technology, then poor countries tend to grow faster than rich countries. Thus, there is a force that promotes convergence in levels of per capita income across countries.

The main element behind the convergence result in neoclassical growth models is diminishing returns to reproducible capital. Poor countries, with low ratios of capital to labor, have high marginal products of capital and thereby tend to grow at high rates. The hypothesis that poor countries tend to grow faster than rich countries seems to be inconsistent with the cross-country evidence, which indicates that per capita growth rates have little correlation with the starting level of per capita product.

The empirical analysis in this paper uses school-enrollment rates as proxies for human capital. For a given starting value of per capita GDP, a country's subsequent growth rate is positively related to these measures of initial human capital. Moreover, given the human-capital variables, subsequent growth is substantially negatively related to the initial level of per capita GDP. Thus, in this modified sense, the data support the convergence hypothesis of neoclassical growth models. A poor country tends to grow faster than a rich country, but only for a given quantity of human capital.”
Figure I: Per capita growth rate (on the vertical scale) versus initial (1960) GDP per capita (in $1000) (on the horizontal axis)

![Graph showing the relationship between per capita growth rate and initial GDP per capita.](image)

Figure II: Partial association between growth rate (on the vertical axis) versus initial per capita income (on the horizontal axis):
Here, the vertical axis shows the average growth rate, net of the value predicted by all explanatory variables in the regression (human capital indicators and other control variables; see next pages) except initial per capita income.

In contrast with Figure I, the relationship is now strongly negative, the correlation is -0.74. Thus the results indicate that, holding constant a set of variables that includes proxies for starting human capital, higher initial per capita GDP growth is substantially negatively related to subsequent per capita growth.
Figure III: Partial association between growth rate (on the vertical axis) versus human capital (on the horizontal axis).
This figure shows the relationship between the per capita growth rate, net of the value predicted by the regressors other than the human capital indicators. The partial correlation between growth rate and the human capital proxy is 0.73, compared with a simple correlation of 0.43.
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<td>—</td>
<td>—</td>
<td>—</td>
<td>−0.0085 (0.0064)</td>
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| g/y              | −0.119 (0.028) | −0.122 (0.028) | −0.142 (0.034) | −0.147 (0.036) | −0.122 (0.032) | −0.106 (0.024) | −0.178 (0.024) | −0.121 (0.027) |
| REV              | −0.0195 (0.0063) | −0.0200 (0.0063) | −0.0236 (0.0071) | −0.0241 (0.0071) | −0.0151 (0.0091) | −0.0192 (0.0067) | −0.0165 (0.0044) | −0.0189 (0.0060) |
| ASSASS           | −0.0333 (0.0155) | −0.0309 (0.0156) | −0.0485 (0.0185) | −0.0490 (0.0188) | −0.0344 (0.0160) | −0.0342 (0.0159) | −0.0241 (0.0271) | −0.0298 (0.0130) |
| PPl60DEV         | −0.0143 (0.0053) | −0.0148 (0.0053) | −0.0171 (0.0078) | −0.0174 (0.0079) | −0.0316 (0.0101) | −0.0237 (0.0069) | −0.0165 (0.0044) | −0.0141 (0.0052) |
| R²               | 0.56 | 0.56 | 0.49 | 0.50 | 0.63 | 0.53 (0.72) | 0.52 (0.84) | 0.56 |
| ϕ                | 0.0128 | 0.0128 | 0.0168 | 0.0189 | 0.0109 | 0.0131 (0.0115) | 0.0133 (0.0120) | 0.0129 |
Human Capital and Growth: Lucas (JME, 1988)

Agents choose how to allocate their time; they can either produce or accumulate human capital (you can either go to school or get a job). Your human capital accumulation today will enhance your productivity tomorrow, but decreases your production today. Therefore, there is a trade-off.

The production function is:

\[ Y = AK^\beta (uhN)^{1-\beta} h_a^\gamma \]

There are \( N \) workers in the economy with \( h \) amount of human capital.

In the above, \( u \) is the share of human capital devoted to goods production. So, \( uhN \) is the total human capital used in the production of the final good.

If \( u \) is the share of human capital devoted to goods production, \( 1-u \) is the share devoted to human capital accumulation. These shares are endogenously determined as a result of the dynamic optimization of agents.

Human capital accumulation occurs according to the following equation:

\[ \dot{h}/h = G(1-u), G'>0, \] where \( G \) is an increasing function of the share of human capital devoted to human capital accumulation.

Lucas assumes that the equation takes a linear form.

\[ \dot{h}(t) = h(t) \delta [1-u(t)]. \]

This means that if no effort is devoted to human capital accumulation, \( u = 1 \) and none accumulates; if all effort is devoted to this pursue, then \( h \) grows at its maximal rate \( \delta \).

In addition to the effects of an individuals’s human capital on his own productivity (the internal effect of human capital), there is the external effect, i.e. the contribution of \( h \) to the productivity of all factors of production.

There are two possible solutions: The optimal path and the equilibrium path. The optimal path emerges as the solution to the social planner’s optimization problem. The equilibrium path emerges as the competitive equilibrium outcome. The two are not the same here, because there is an external effect of human capital accumulation. The agents take the term \( h_a^\gamma \) as given when optimizing, but for the social planner this term is a choice variable. The planner internalizes the external effect of human capital accumulation.

In this model, if \( \gamma = 0 \) and therefore \( h_a^\gamma = 1 \), the two paths are the same. When \( \gamma > 0 \), this creates a divergence between the social valuation and private valuation of human capital. The social value is higher, but agents take into account only the private return when making their decisions.

The model can be solved for the long-run growth rates of consumption, human capital and physical capital, which are all positive and determined by the parameters of the model, such as \( \gamma, \beta, \delta \) and the time preference rate of consumers (not shown here). Therefore, we conclude that with human capital accumulation along with physical capital accumulation, a
positive rate of per capita growth is achievable. In fact, Lucas shows that positive growth rates are possible whether or not the external effect is positive.

**Technical Progress and Growth:**

Remember that in the Solow model all long-run per capita growth comes from technical progress (the rate at which the productivity of factors increases). In endogenous growth models, there are other sources of growth, such as savings and human capital. In these types of models, we have assumed production exhibits constant returns to all deliberately accumulated inputs.

As soon as we accept the existence of some fixed factor of production (and the overall CRS), continued per capita growth gets hard to explain without relying on increases in the body of knowledge. The reason is that diminishing returns set in if the per capita magnitudes of accumulated factors become too large relative to the fixed factor.

Technical progress does not occur in a vacuum. R&D activities include many researchers hired by profit-seeking firms. There are roughly two kinds of technical progress:

a) Deliberate diversion of resources from consumption to knowledge production in the hope of making profit in the future. These include the introduction of new products (product innovation) and new methods (process innovation).

b) Transfer of technical knowledge from the innovating firm to the rest of the world. This can happen in two ways. The new technology may become known to others who profit from it directly, or the new technology may lead to innovative activity by others.

The two types have very different implications. The first type fits the profit motive of an innovator who will invest in the R&D activity with the expectation of reaping the benefits of his investment. Patent protection becomes crucial right away. The second type gives us the impression that faster diffusion of technical progress will lead to more technical progress; however the story is more complicated. Diffusion may slow down the rate of deliberate technical progress, or it may spur more innovation as leaders of the technology struggle to stay ahead of their rivals.

**A model of deliberate technical progress (Romer, 1990)**

Assume that there is a fixed supply of human capital (H), which can be used in the production of either the final goods or knowledge. Production is carried out by machines and labor (including skilled labor). We already know how to produce some of the machines (we have their blueprints), however we have to devote resources to discover how to produce new machines.

Let \( E_t \) denote the amount of technical knowledge at time \( t \). The growth rate of knowledge is expressed as \( \frac{E_{t+1} - E_t}{E_t} = a(1-u)H \), where \( a \) is a constant, \( u \) is the share of \( H \) devoted to the production of final goods, and \( (1-u) \) is the share of \( H \) devoted to the production of knowledge.
Assume that once we know how to produce a machine, we can produce it by using 1 unit of capital.

The R&D (knowledge production) activity contributes to productivity in two ways. New machines may be more productive than the older ones and thus replace them, or it may be the case that increasing variety of machines lead to higher productivity (as in the case of fax machine, telephone, e-mail etc.).

The production of final goods takes place according to the following production function:

$$ Y_t = E_t^\gamma K_t^\alpha (uH)^{1-\alpha}, $$

where $E_t^\gamma K_t^\alpha$ denotes the combined effect of the total stock of machines and their productivities.

Since one machine can be produced by one unit of capital, the total quantity of capital can be thought of as the total stock of machines available for production.

Capital evolves according to the familiar equation,

$$ K_{t+1} = K_t + sY_t. $$

Note that in this model, unlike the Solow model, technical progress is not exogenous, but is determined by $H$ and its utilization rate in the R&D sector.

How is $u$ chosen?

We need to consider the trade-off between using $H$ for producing more final goods today and using it in the R&D sector for more final goods tomorrow. Moreover, we need to consider the profit motive of the private sector. The blueprints, once produced, can be reproduced almost costlessly. (Remember the properties of public goods.) For this reason, no innovator would produce blueprints without a monopoly power (at least temporary) in the market for his innovation through patent protection. Therefore, the choice of $u$ will depend on factors such as the degree of appropriability of the technology through patent protection and the rate of diffusion of knowledge to others.

A model of externalities and technical progress (Romer, 1986)

Remember what an externality is?

Suppose an industrialist build a railway line that connects a mining center to a port city and passes through a sleepy town.

The railway benefits the town. (For instance, increased transportation opportunities cause some people to move to this town and commute to the city to work. Real estate prices go up.) Suppose that the industrialist is unable to charge the residents for the extra benefit that they are receiving. There is a positive externality here.

The railway passes through another town, but does not stop there. The residents have to bear with the additional noise and pollution. They can not charge the industrialist who built the railway. There is a negative externality here.
Capital accumulation and technological progress create externalities, generally positive. How do we model this? Imagine that there are several firms in an economy and these firms have the following production function:

\[ Y_t = E_t K_t^\alpha P_t^{1-\alpha}, \]

where \( E_t \) is a measure of overall productivity and it is a parameter common to all firms in the economy.

Now suppose that the productivity parameter is neither exogenously specified (as in the Solow model) nor determined by deliberate R&D activity (as in the previous model), but is a positive externality generated by the joint capital accumulation of all firms in the economy. In other words, capital accumulation by each firm is based on selfish decisions, however these decisions have positive effects on all other firms.

In this case, the productivity parameter can be written as a function of the average capital stock in the economy.

\[ E_t = a(K_t^*)^{\beta}. \]

(a and \( \beta \) are positive constants.)

The production function of a particular firm can be written as

\[ Y_t = a(K_t^*)^{\beta} K_t^\alpha P_t^{1-\alpha}. \]

Here, we can assume that firms learn by doing, i.e. by investing and producing. Also assume that each firm’s knowledge is a public good that any other firm can access at zero cost. In other words, knowledge spills over across the entire economy. Here, discoveries are unintended by-products of investment and that these discoveries immediately become common knowledge. These assumptions allow us to keep the perfect competition framework.

In this model, no firm values the positive externality that it has on other firms. The reason is simple: There is no way of charging other firms for the benefits they receive. The private marginal benefit from investing is less than the social marginal benefit. For this reason firms underinvest in capital accumulation, relative to the case in which they would receive the full benefits of their investment. The solution is to internalize the externality, which can be done by a benevolent planner who owns all of the firms in the economy.

Another implication of positive externalities is that CRS at the firm level can coexist with IRS (increasing returns to scale) at the economy level. Notice that each firm faces diminishing returns to its own capital. To see the increasing returns to scale, assume for simplicity that all firms are identical, that is to say \( K_t = K_t^* \). In this case, the “macroeconomic” production function can be written as:

\[ Y_t = a(K_t^*)^{\beta} K_t^\alpha P_t^{1-\alpha} = aK_t^{\alpha + \beta} P_t^{1-\alpha}, \]

which exhibits IRS.

It has been shown that with IRS, per capita growth is not just positive, but tends to accelerate in the long-run!!

Competitive equilibrium growth rate is lower than the rate at social optimum (when the externality is internalized) with a benevolent social planner. The reason is that the firms do not internalize the benefits of spillovers; they base their decisions on the private marginal product, which falls short of social marginal product.
Social optimum can be achieved in a decentralized economy by subsidizing capital accumulation or production so as to raise the private return to investment.

**Complementarities:**

Complementarities are a special type of externality. Externalities refer to the level of satisfaction (or reward or punishment) experienced by others as the result of your actions. Complementarities refer to an increased relative preference that others experience for choosing similar actions because you acted in a particular way.

For example, by driving safely you inflict a positive externality on others, but this does not induce others to drive safely. In the case of the railway, the ones who benefit from the railway have no reason to act in a similar way, i.e. to build railway lines themselves. But in the “externalities and technical progress” model, the investment decision by one firm raises other firms’ incentives to do so as well. In this model, each firm chooses its own saving (and thus investment) rate s. If the firm believes that the average s in the economy is high, then so will be its expectation of its own productivity and it will choose a larger s.

![Graph showing individual (actual) investment rates vs. anticipated average investment rate](image)

Note that the average investment rate is not exogenous, but is generated by the actions of all firms in the economy. Assume for simplicity that all firms are identical. In the figure above, the curve shows the investment rate of a firm as a function of its expectation of the average investment rate in the economy. The intersections of this curve with the 45-degree line are the equilibrium points, i.e. points at which the expectations match actual outcomes. If all firms believe that a low investment rate such as s1 will prevail, then all of these firms will invest at rate s1, thus justifying the prior beliefs. On the other hand, if all firms are optimistic about investment, then they will all invest at a high rate such as s2, again justifying their beliefs. A point such as s3 can not be an equilibrium.

The possibility of multiple equilibria when there are complementarities suggests that two identical copies of the same economy might behave differently and grow at different rates, depending on how historical experiences shape expectations.
Total factor productivity

How can we measure technical progress?
Let’s write the production function as:

\[ Y_t = F(K_t, P_t, E_t) \]

We can obtain estimates of \( Y, K \) and \( P \), although there are important measurement problems. But it is impossible to directly measure \( E \), which represents the state of knowledge. Therefore, we will need a trick to obtain an estimate of \( E \), or at least the growth in output that is attributable to \( E \).

Assuming that there is no change in \( E \), the change in output from time \( t \) to time \( t+1 \) can be written as:

\[ \Delta Y_t = \Delta MPK_K + \Delta MPL_L \]

where \( MPK \) is the marginal product of capital and \( MPL \) is the marginal product of labor.

Dividing by \( Y_t \) and with some manipulation, we obtain:

\[ \frac{\Delta Y_t}{Y_t} = \frac{\Delta MPK_K}{K_t} + \frac{\Delta MPL_L}{P_t} \]

Under the assumptions of CRS and competitive markets, factors are paid their marginal products. Therefore, \( MPK_K \) and \( MPL_L \) represent the total payments to capital and labor, respectively. Dividing these quantities by total output gives us the income shares of capital and labor in national income, \( \Phi_K \) and \( \Phi_P \). The equation can be written as:

\[ \frac{\Delta Y_t}{Y_t} = \Phi_K \frac{\Delta K_t}{K_t} + \Phi_P \frac{\Delta P_t}{P_t} + \text{TFPG}_t \]

Note that everything in the above equation is measurable by looking at the data. If the two sides of the equation are equal, then we can infer that the change in output can be explained by changes in capital and labor, and that \( E \), the third factor of production, has not changed. If the two sides are not equal, then there must have been a change in \( E \). The difference between the left-hand side of the equation and the right-hand side is the growth in total factor productivity (TFP) between time \( t \) and \( t+1 \), denoted by \( \text{TFPG}_t \).

\[ \frac{\Delta Y_t}{Y_t} = \Phi_K \frac{\Delta K_t}{K_t} + \Phi_P \frac{\Delta P_t}{P_t} + \text{TFPG}_t \]

Therefore, \( \text{TFPG}_t \) is positive when output is increasing faster than predicted by the growth in inputs. This is a way to quantify technical progress as the residual in the growth equation.

Some points to note:
1. You might have noticed that the level of total factor productivity is not important, since it can be chosen arbitrarily. What matters is the growth in TFP, and it can be calculated by using the method described above.
2. The standard method to measure \( \Delta P_t/P_t \) is to use the population growth rate. But this might be misleading when labor force participation rate is changing fast.
3. Problems arise in the measurement of capital, labor and their changes over time, since total capital and total labor are made up of different categories growing at different rates.
4. The methodology fails if the factors of production are not paid their marginal products, or if the production function is not CRS.

Total factor productivity and the East Asian miracle:

Over the period 1965-90 the region (Japan, Hong Kong, Korea, Taiwan, Singapore, Indonesia, Thailand and Malaysia) grew faster than any other region in the history of the world. These countries are known for their high saving rates and skilled labor force. What was the main source of growth in these countries? Capital (human and physical) accumulation or technical progress?

The answer has important implications for policy. If it is mainly capital accumulation that creates growth, then this is what the government should encourage. On the other hand, if it is mainly technical progress that creates growth, then the government should follow policies that favor sectors conducive to technological innovation or assimilation. The study titled “the East Asian Miracle” by the World Bank finds that approximately two-thirds of the observed growth in these economies can be attributed to physical and human capital accumulation and the remaining one-third to TFP growth. The study does not argue that TFP growth is the dominant factor, but relative to other developing countries its contribution appears to be very high indeed. This study then goes on to argue that openness to trade enabled exports to help these economies master international best-practice technologies. High levels of human capital in these countries helped firms better adopt and master the technology. Thus, exports and human capital interacted to provide a “rapid phase of productivity-based catching up”.

We mentioned before that there are substantial problems in the measurement of changes in inputs. One needs to account for rising participation rates, transfer of labor between agriculture and industry and changes in the education levels. Young (1995) does just this. The following table reports his findings and lists TFP growth rate in several other countries for comparison.

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Growth</th>
<th>Country</th>
<th>Period</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong Kong</td>
<td>1966-90</td>
<td>2.3</td>
<td>Brazil</td>
<td>1950-85</td>
<td>1.6</td>
</tr>
<tr>
<td>Singapore</td>
<td>1970-90</td>
<td>0.2</td>
<td>Mexico</td>
<td>1940-85</td>
<td>1.2</td>
</tr>
<tr>
<td>South Korea</td>
<td>1966-90</td>
<td>1.7</td>
<td>United Kingdom</td>
<td>1960-89</td>
<td>1.3</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1966-90</td>
<td>2.1</td>
<td>Venezuela</td>
<td>1950-70</td>
<td>2.6</td>
</tr>
<tr>
<td>France</td>
<td>1960-89</td>
<td>1.5</td>
<td>Japan</td>
<td>1960-89</td>
<td>2</td>
</tr>
<tr>
<td>Germany</td>
<td>1960-89</td>
<td>1.6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average TFP growth rates in selected countries

It is true that East Asian countries have grown rapidly, but it appears that they have grown the traditional way: via factor accumulation, i.e. through an extraordinary process of labor force improvement and sustained capital accumulation. In other words, technological improvement was not the most important factor. As can be seen in the table, it is not difficult to find either developed or developing countries whose productivity performance, despite considerably slower output per capita growth, has approximated or matched that of the East Asian economies. While productivity growth in these countries is not particularly low, it is not extraordinarily high by postwar standards.
Exercise: (From Weil’s textbook, Economic Growth)

Differences in the level of productivity among countries. How much do productivity differences explain income differences?

Let \( y = Ak^\alpha h^{1-\alpha} \)

Take two countries, look at the ratio of their per capita incomes.

\[
\frac{y_1}{y_2} = \left( \frac{A_1}{A_2} \right) \left( \frac{k_1^\alpha h_1^{1-\alpha}}{k_2^\alpha h_2^{1-\alpha}} \right)
\]

\[
\frac{A_1}{A_2} = \left( \frac{y_1}{y_2} \right) \left( \frac{k_2^\alpha h_2^{1-\alpha}}{k_1^\alpha h_1^{1-\alpha}} \right)
\]

The following table presents the values of variables in several countries relative to the values in the US. For example, in column 2, you see the ratio of \( y \) in a country to \( y \) in the US.

There are surprisingly large differences in the level of productivity across countries. For example, if Korea and the US had the same \( k \) and \( h \), the Korea would produce only 63% as much \( y \) as the US.

<table>
<thead>
<tr>
<th>Country</th>
<th>Output per Worker, ( y )</th>
<th>Physical Capital per Worker, ( k )</th>
<th>Human Capital per Worker, ( h )</th>
<th>Factors of Production, ( k^{1/3}h^{2/3} )</th>
<th>Productivity, ( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Norway</td>
<td>0.92</td>
<td>1.08</td>
<td>0.97</td>
<td>1.01</td>
<td>0.92</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.76</td>
<td>0.69</td>
<td>0.97</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Canada</td>
<td>0.75</td>
<td>0.86</td>
<td>1.01</td>
<td>0.96</td>
<td>0.79</td>
</tr>
<tr>
<td>Japan</td>
<td>0.69</td>
<td>1.10</td>
<td>0.99</td>
<td>1.02</td>
<td>0.67</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.54</td>
<td>0.73</td>
<td>0.93</td>
<td>0.86</td>
<td>0.63</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.29</td>
<td>0.27</td>
<td>0.79</td>
<td>0.56</td>
<td>0.52</td>
</tr>
<tr>
<td>Peru</td>
<td>0.14</td>
<td>0.12</td>
<td>0.82</td>
<td>0.44</td>
<td>0.32</td>
</tr>
<tr>
<td>India</td>
<td>0.13</td>
<td>0.10</td>
<td>0.74</td>
<td>0.38</td>
<td>0.35</td>
</tr>
<tr>
<td>Cameroon</td>
<td>0.10</td>
<td>0.036</td>
<td>0.58</td>
<td>0.23</td>
<td>0.44</td>
</tr>
<tr>
<td>Zambia</td>
<td>0.034</td>
<td>0.032</td>
<td>0.65</td>
<td>0.24</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Sources: Output per worker: Heston, Summers, and Aten (2006); physical capital: author’s calculations; human capital: Cohen and Soto (2007). The data set used here and in Section 7.3 is composed of data for 78 countries for which consistent data are available for 1970 and 2005.