Exercises: Sampling Distributions

$\bar{x}$	$P(\bar{x})$
1	1/36
1.5	2/36
2	3/36
2.5	4/36
3	5/36
3.5	6/36

6.1 Suppose that you toss a pair of dice and write down the value of the faces from each die.

a. What is the population distribution for one die?
b. Determine the sampling distribution of the sample means obtained by tossing two dice.

6.6 Given a population with a mean of $\mu = 100$ and a variance of $\sigma^2 = 900$, the central limit applies when the sample size $n \geq 25$. A random sample of size $n = 30$ is obtained.

a. What are the mean and variance of the sampling distribution for the sample means?
b. What is the probability that $\bar{x} > 109$?
c. What is the probability that $96 \leq \bar{x} \leq 110$?
d. What is the probability that $\bar{x} \leq 107$?

a) $\mu_{\bar{x}} = \mu_x = 100$, $\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{900}{30} = 30$

b) $P(\bar{x} > 109) = P\left(z > \frac{109-100}{\sqrt{30}}\right) = P(z > 1.64)$

$= 0.505$

c) $P(96 < \bar{x} < 110)$

$= P(-0.73 < z < 1.83) = 0.7337$

d) $= P(\bar{x} < 107) = P(z < 1.28) = 0.8997$

6.9 When a production process is operating correctly, the number of units produced per hour has a normal distribution with a mean of 92.0 and a standard deviation of 3.6. A random sample of 4 different hours was taken.

a. Find the mean of the sampling distribution of the sample means.
b. Find the variance of the sample mean.
c. Find the standard error of the sample mean.
d. What is the probability that the sample mean exceeds 93.0 units?

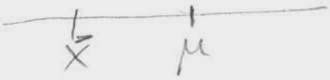
$X \sim N(92, \sigma^2 = 3.6^2)$

a) $\mu_{\bar{x}} = 92$

b) $\sigma^2_{\bar{x}} = \frac{3.6^2}{4}$, c) $\sigma_{\bar{x}} = \frac{3.6}{2} = 1.8$

d) $P(\bar{x} > 93) = P\left(z > \frac{93-92}{1.8}\right)$

$= P(z > 0.56) = 0.2877$



6.17 The times spent studying by students in the week before final exams follows a normal distribution with standard deviation 8 hours. A random sample of four students was taken in order to estimate the mean study time for the population of all students.

a. What is the probability that the sample mean exceeds the population mean by more than 2 hours?
b. What is the probability that the sample mean is more than 3 hours below the population mean?
c. What is the probability that the sample mean differs from the population mean by more than 4 hours?
d. Suppose that a second (independent) random sample of 10 students was taken. Without doing the calculations, state whether the probabilities in parts (a), (b), and (c) would be higher, lower, or the same for the second sample.

$\sigma_{\bar{x}} = 8/\sqrt{4} = 4$

a) $P(\bar{x}-\mu>2) = P(z>2/4) = 0.3085$

b) $P(\mu-\bar{x}>3) = P(\bar{x}-\mu<-3)$
$= P(z<-3/4) = 0.2266$

c) $P(|\bar{x}-\mu|>4)$
$= P(\bar{x}-\mu>4) + P(\mu-\bar{x}>4)$
$= P(z>1) + P(-z>1)$
$= P(z>1) + P(z<-1)$
$= 0.3174$

d) Lower, lower, lower

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6.19 The price-earnings ratios for all companies whose shares are traded on the New York Stock Exchange follow a normal distribution with a standard deviation 3.8. A random sample of these companies is selected in order to estimate the population mean price-earnings ratio.

a. How large a sample is necessary in order to ensure that the probability that the sample mean differs from the population mean by more than 1.0 is less than 0.10?

b. Without doing the calculations, state whether a larger or smaller sample size compared to the sample size in part (a) would be required to guarantee that the probability of the sample mean differing from the population mean by more than 1.0 is less than 0.05.

c. Without doing the calculations, state whether a larger or smaller sample size compared to the sample size in part (a) would be required to guarantee that the probability of the sample mean differing from the population mean by more than 1.5 hours is less than 0.05.

$\sigma = 3.8$ $\quad \sigma_{\bar{x}} = 3.8/\sqrt{n}$

$P(|\bar{x}-\mu|<1) \geq 0.90$

From the table,

$P(|z|>1.645) = 10\%$

$\frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = 1.645 \Rightarrow (1.645)\frac{3.8}{\sqrt{n}} = 1$

$\Rightarrow n = 39{,}075 \Rightarrow n \cong 40$ (at least)

$\frac{1.96(3.8)}{1.0} = \sqrt{n}$

$\Rightarrow n = 55{,}47$

$n \approx 56$

b) Larger, $n \cong 56$

c) $\%2.5 = 0.025$

$1 - 0.025 = 0.9750$

$P(z < 1.96) = 0.9750$

$P(|z| > 1.96) = 5\%$

$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = 1.96 \Rightarrow$

$1.5\sqrt{n} = (1.96)(3.8)$

$\sqrt{n} = 4.965 \Rightarrow n = 24.65$

$n \cong 25$

$\frac{(1.645)(3.8)}{1.5} = \sqrt{n} \leftarrow 0.10$

$\Rightarrow n = 17.36, \; n \cong 18$ smaller

6.26 Suppose that we have a population with proportion $P = 0.40$ and a random sample of size $n = 100$ drawn from the population.

a. What is the probability that the sample proportion is greater than 0.45?

b. What is the probability that the sample proportion is less than 0.29?

c. What is the probability that the sample proportion is between 0.35 and 0.51?

$E(\hat{p}) = 0.4 \quad \sigma_{\hat{p}} = \left(\frac{(0.4)(0.6)}{100}\right)^{1/2}$

a) $P(\hat{p} > 0.45) = P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} > 1.02\right)$ $= 0.1539$

b) $P(\hat{p} < 0.29) = P(z < -2.25) = 0.0122$

c) $P(0.35 < \hat{p} < 0.51) =$

$P(-1.02 < z < 2.25) = 0.8339$

6.35 A charity has found that 42% of all donors from last year will donate again this year. A random sample of 300 donors from last year was taken.

a. What is the standard error of the sample proportion who will donate again this year?

b. What is the probability that more than half of these sample members will donate again this year?

c. What is the probability that the sample proportion is between 0.40 and 0.45?

d. Without doing the calculations, state in which of the following ranges the sample proportion is more likely to lie: 0.39 to 0.41, 0.41 to 0.43, 0.43 to 0.45, or 0.45 to 0.46.

a) $\sigma_{\hat{p}} = \sqrt{\frac{(0.42)(0.58)}{300}} = 0.0285$

b) $P\left(z > \frac{0.5-0.42}{\sigma_{\hat{p}}}\right) = P(z > 2.81)$

$= 0.0025$

c) 0.6111

d)

6.42 Suppose that 50% of all adult Americans believe that federal budget deficits at recent levels cause long-term harm to the nation's economy. What is the probability that more than 58% of a random sample of 250 adult Americans would hold this belief.

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.5)(0.5)}{250}} = 0.03162$$

$$P\left(z > \frac{0.58-0.5}{0.03162}\right) = P(z>2.53)$$
$$= 0.0057$$

6.51 Monthly rates of return on the shares of a particular common stock are independent of one another and normally distributed with a standard deviation of 1.6. A sample of 12 months is taken.

a. Find the probability that the sample standard deviation is less than 2.5.

b. Find the probability that the sample standard deviation is more than 1.0.

$$a)\ P\left(\frac{(n-1)s^2}{\sigma^2} < \frac{\overset{68.75}{11(2.5)^2}}{\underset{2.56}{(1.6)^2}}\right) = P(\chi^2_{11} < 26.85)$$
$$\geq 0.995$$

$$b)\ P\left(\chi^2_{11} > \frac{11(1)}{1.6^2}\right) = P(\chi^2_{11} > 4.296)$$

bw 0.975 and 0.950

6.57 A production process manufactures electronic components with timing signals whose duration follows a normal distribution. A random sample of six components was taken, and the durations of their timing signals were measured.

a. The probability is 0.05 that the sample variance is bigger than what percentage of the population variance?

b. The probability is 0.10 that the sample variance is less than what percentage of the population variance?

$$a)\ P(\chi^2_5 > 11.07) = 0.05$$
$$P(s^2 > x\sigma^2) = P\left(\frac{(n-1)s^2}{\sigma^2} > 5x\right)$$
$$11.07 = 5x \Rightarrow x = 2.214$$
$$x = 221.4\%$$

$$b)\ P(s^2 < x\sigma^2) = 0.10$$
$$P(\chi^2_5 < 1.61) = 0.10$$
$$P(s^2 < x\sigma^2) = P\left(\frac{(n-1)s^2}{\sigma^2} < (n-1)x\right)$$
$$= P(\chi^2_5 < 5x) = 0.10$$
$$1.61 = 5x$$
$$\Rightarrow x = 0.322$$
$$\Rightarrow x = 32.2\%$$

6.61 A drug company produces pills containing an active ingredient. The company is concerned about the mean weight of this ingredient per pill, but it also requires that the variance (in squared milligrams) be no more than 1.5. A random sample of 20 pills is selected, and the sample variance is found to be 2.05. How likely is it that a sample variance this high or higher would be found if the population variance is, in fact, 1.5? Assume that the population distribution is normal.

$n = 20$, $s^2 = 2.05$

$$P(s^2 > 2.05) = P\left(\frac{(n-1)s^2}{\sigma^2} > \frac{(19)(2.05)}{1.5}\right) = P(\chi^2_{19} > 25.96) > 0.10.$$

%13.10 exactly

$$b)\ P(s^2 > 3) = ?$$
$$P\left(\chi^2_{19} > \frac{(19).3}{1.5}\right) = P(\chi^2_{19} > 38)$$ bw 0.01 and 0.005

Would you doubt that σ^2 is indeed 1.5 ?