Chapter 3: Economic Growth

Quoting Robert Lucas,
"Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what, exactly? If not, what is it about the nature of India that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts thinking about them, it is hard to think about anything else."

Fact: Economic growth is a modern phenomenon. Appreciable per capita income growth was the exception rather than the rule.

For example, during 1580-1820, Netherlands was the leading industrial nation and it experienced an annual growth in real GDP per worker hour of 0.2%.

During 1890-1989, US experienced a growth rate of (much higher) 2.2% per year.

Keep in mind that small changes in growth rate can make a huge difference in the long run. Remember the 70/g rule (an approximation). At 2% average annual rate of growth, incomes will double in about 70/2=35 years. At 1% growth rate, doubling will take 70 years.

Taking a look at the historical experiences of countries, a century ago the now-developed countries grew in a world in which nations of far greater economic strength did not exist. Today, developing nations not only need to grow, but must grow faster than before. The developed countries of the world are highly visible and the power they exercise is evident. With the increasing flow of information, people all around the world are increasingly and more quickly aware of new products and higher standards of living. The increased perception of global inequalities makes the need for sustained growth all the more urgent.

The Harrod-Domar Model:

Assume that:
- the economy is closed.
- there is unemployed labor (the scarce resource is capital.)
- production is proportional to capital stock; $\theta = K_t / Y_t$, see below.

$Y_t = C_t + S_t$ (income is either consumed or saved)
The other side of the coin:
$Y_t = C_t + I_t$ (the total value of produced output is the sum of consumption goods and capital goods produced)
Combining the two; we get $S_t = I_t$

Investment augments the national capital stock (K) and replaces the depreciating part of the capital stock. Assume that δ is the fraction of the capital stock that depreciates each period. Then; $K_{t+1} = (1- \delta)K_t + I_t$

Define saving rate as: $s = S_t / Y_t$
Define capital-output ratio as: $\theta = K_t / Y_t$

We can write $K_{t+1} = (1-\delta)K_t + I_t$
or $\theta Y_{t+1} = (1-\delta)\theta Y_t + sY_t$.
Rewriting; $\theta Y_{t+1} - \theta Y_t = sY_t - \delta\theta Y_t$
or

$$\frac{Y_{t+1} - Y_t}{Y_t} = \frac{s}{\theta} - \delta$$

Define the growth rate as: $g = (Y_{t+1} - Y_t)/ Y_t$

This gives us the main equation of the model: $$\frac{s}{\theta} = g + \delta$$

This equation links the growth rate of total income to two fundamental variables: s and θ. It appears that it would be possible to accelerate the rate of growth by increasing the saving rate and/or by increasing the rate at which capital produces output (thereby lowering θ).

We now incorporate the effects of population growth.

Define per capita income as $y_t = Y_t / P_t$
Dividing both sides of $\theta Y_{t+1} = (1-\delta)\theta Y_t + sY_t$ by P_t yields:

$$\frac{\theta\ Y_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} = (1-\delta)\theta \frac{Y_t}{P_t} + s\frac{Y_t}{P_t},\ \text{or}$$

$$\theta\ y_{t+1} \frac{P_{t+1}}{P_t} = (1-\delta)\theta\ y_t + sy_t$$

Dividing both sides by $y_t \theta$, and noting that $y_{t+1} / y_t = 1+g^*$, where g^* is the growth rate of per capita income, and that $P_{t+1} / P_t = 1+n$, where n is the population growth rate, we obtain:

$$\frac{s}{\theta} = (1+g^*)(1+n) - (1-\delta),$$

or, since g^* and n are usually small numbers, as an approximation, $\frac{s}{\theta} \cong g^* + n + \delta$.

This equation says that if savings rates, capital-output ratios, population growth rates and depreciation rates are such and such, then per capita income growth rate is g^* percentage points.

There are some problems with this argument:

- Investment is not a simple aggregate object, its composition is important,
- The parameters that are used to predict the growth rate may themselves be a function of the growth rate (i.e. endogeneity)

What are different sources of endogeneity?

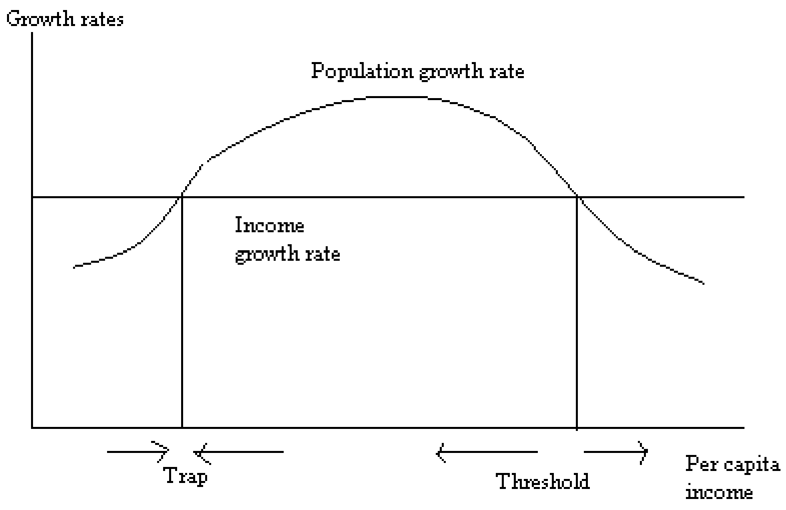
- Saving rate may depend on income, that is we may need to change the saving rate that enters the formula as incomes change:

At low levels of income, people usually do not have the ability to save and it is unlikely that the policy maker can increase the saving rate. At this stage, growth needs external credit. As incomes increase, saving rate increases, but not necessarily. As incomes grow, needs may grow as well. The existence of some inequality in income may help increase the savings of the middle class, who aspire to reach a higher rank in the income distribution later by saving more today. As incomes go up further, what will happen to the saving rate is ambiguous. The rich are certainly able to save, at least at the same rate as the middle class, but since they are already rich, current consumption might be more attractive to them.

- Population growth rate may depend on income: (Demographic transition) In poor countries, death rates among children are high. Families, therefore, choose to have many children in order to reach a target number of surviving children. As the country gets richer, with improving life standards, death rates fall, but birth rates adjust slowly. This causes population growth rate to increase. With further development, birth rates begin their downward adjustment and population growth rate falls again.

In the diagram below, the horizontal line shows the growth rate of income, assumed to be constant for simplicity. The curve shows the population growth rate. When population growth rate is below income growth rate, per capita income rises. When the opposite is true, per capita income falls. This creates a “Trap” which inhibits an increase in per capita income. Even if a country has a per capita income that is greater than the “Trap” level, since population growth rate is higher than income growth rate, per capita income is decreasing.

If a country is fortunate enough to be to the right of the “Threshold”, per capita income will increase over time. This suggests that a *temporary* boost to certain parameters through policy change may have sustained long-run effects. For example, a jump in the saving rate may shift the growth rate of income up so that the threshold is passed. Or, a family planning education program may bring down the population growth rate and allow the country to pass the threshold.



- From the book titled "the Elusive Quest for Growth" by William Easterly, 2001, Ch. 2:

In April 1946, economics professor Evsey Domar published an article on economic growth, "Capital Expansion, Rate of Growth and Employment" which discussed the relationship between short-term recessions and investment in the United States. Although Domar assumed that production capacity was proportional to the stock of machinery, he admitted the assumption was unrealistic and eleven years later he said his earlier purpose was to comment on business cycles, not to derive "an economically meaningful rate of growth". He said his theory made no sense for long-run growth.

Ironically, this model became the most widely applied growth model in economic history. We economists applied it to poor countries to determine the "required" investment rate for a target growth rate. The difference between the required rate and the country's own savings was called the *financing gap*. It was thought to be necessary that donors fill this gap with aid to attain target growth. Therefore, the Harrod-Domar model turned into a model that promised poor countries growth through aid-financed investment. Aid would lead to investment which would lead to growth.

In 1960, Rostow published "The Stages of Economic Growth, A Non-Communist Manifesto". Of the five stages Rostow projected, the most well-known is the "takeoff into self-sustained development". The only determinant of output takeoff was investment. Rostow tried to show that the investment-led takeoff idea fit the stylized facts. Stalin's Russia influenced him a lot. However the evidence he cited was weak. Only three of fifteen cases he cited fit the story.

Regardless of the evidence, Rostow's stages drew a great deal of attention to the poor nations. Rostow was the most important advocate for foreign aid. He played on cold war fears. He saw in Russia, a nation surging under communism into a status as an industrial power of the first order. Other observers noted the country's willingness to extract large savings. They warned that the country derived certain advantages from the centralized character of the operation. There was danger that the Third World would go communist.

Rostow felt the need to show to the Third World that communism was not the only form of effective state organization that can launch a takeoff. Western nations could provide them with aid to fill the financing gap between the necessary investment and the actual national saving.

Rostow was in government through the administrations of Kennedy and Johnson. Under both administrations foreign aid increased substantially.

Unfortunately, foreign aid rarely helped Third World countries raise their growth rates, mainly because it provided them with the wrong kind of incentives. Think of the incentives facing the recipients of foreign aid. They invest in the future when they get a high return to their investments. There is no reason to think that the aid given changes the incentives to invest in the future. If the incentives for investment do not change, aid will probably be used by the recipients to consume more. When we empirically check the aid-investment relationship, we see that on balance there is no relationship.

- *From Chapter 3 in the same book:*

Solow’s conclusion surprised many: investment in machinery cannot be a source of growth in the long run. Solow argued that the only possible source of growth in the long run is technological change.

In this model, as we increase machines per worker, each worker will be using more and more machines in time. It’s hard to believe that anything good will happen when we give one more machine to a worker who already has eight. This is called diminishing returns.

Imagine that you are making pancakes by combining ready-to-use pancake mix with milk. Increasing one ingredient (milk) while keeping the other fixed will not enable me to achieve sustained growth in pancake production. Diminishing returns eventually set in.

How severe diminishing returns are depends on the importance of capital in production. In pancake production, salt is a very minor ingredient. If I kept it constant while increasing the amounts of pancake mix and milk, I would probably be fine. But keeping the major ingredients constant while increasing the amount of a minor ingredient like salt is a lot more problematic; i.e. diminishing returns set in much more quickly. So we need to know how important capital is in the production function.

Capital accounts for roughly 1/3 of total production. It was a surprise to economists that buildings and machines are such a minor ingredient in total GDP. With such a small share, diminishing returns to investment will be severe. When machines are scarce, the additional output of one more machine will be high; when machines are abundant, additional output from one more machine will be low.

The Solow Model:

This model is based on the law of diminishing returns to individual factors of production. Unlike the Harrod-Domar model, here the capital-output ratio, θ, is endogenous.

As before, we have,
$S_t = I_t$ and
$K_{t+1} = (1- \delta)K_t + I_t$.

Using $I_t = S_t = sY_t$, we can write $K_{t+1} = (1-\delta)K_t + sY_t$.
Now, express the above in terms of per capita magnitudes, i.e. divide by P_t. Assume that population grows at a constant rate, $P_{t+1} / P_t = 1+n$.

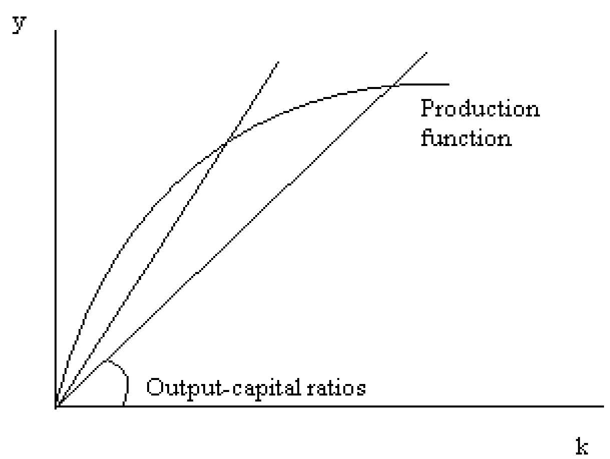
$$k_{t+1} = \frac{(1-\delta)k_t + sy_t}{(1+n)}$$ (Lowercases represent per capita magnitudes.)

This equation says that next period's per capita capital stock is equal to the sum of this period's per capita capital stock, adjusted by depreciation, and this period's per capita savings, divided by a factor that has population growth rate n in it.

$$n\uparrow,\ \ k_{t+1}\downarrow$$
$$s\uparrow,\ \ k_{t+1}\uparrow$$
$$\delta\uparrow,\ \ k_{t+1}\downarrow$$

We can write the law-of-motion equation of this model as follows:
$(1+n)k_{t+1} = (1-\delta)k_t + sf(k_t)$, where f(k) is a CRS production function.

One property of this model, as mentioned before, is the endogeneity of the capital-output ratio, θ. We can show this on a diagram.

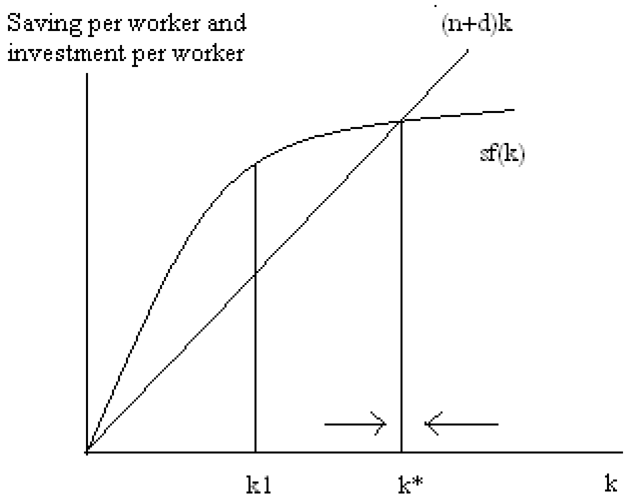


At steady-state, $k_{t+1} = k_t = k^*$ so that:

$$(1+n)k^* = (1-\delta)k^* + sf(k^*),$$

or $$(n+\delta)k^* = sf(k^*).$$

(From “Macroeconomics” by Abel and Bernanke)
This equation says that at equilibrium, savings per worker, sf(k), is equal to investment per worker, (n+δ)k, the latter being the amount of investment per worker required to replace depreciating capital and equip new workers with the same level of capital.
From any starting point, the capital-labor ratio, k, eventually reaches k*, i.e. there is a unique steady-state. When k= k*, the amount that people choose to save will just equal the amount of investment necessary to keep capital per worker at k*. Thus, when the economy’s k reaches k*, it will remain there forever.
If initially $k_1 < k^*$, then savings per worker exceed investment per worker needed to keep k at k_1. The extra saving is converted to capital, and k increases. If, on the other hand, initially k is greater than k*, then saving is too low to keep k at the initial level, therefore k decreases.
To summarize, if there is no productivity growth, the economy reaches a steady-state where k, y and consumption per worker $c = f(k) - (n+\delta)k$ remain constant over time.



In this model, growth slows down if capital stock is increasing too fast relative to labor. The reason for this is diminishing returns, the main assumption of the model. As k increases, output-capital ratio (the inverse of θ) declines, which brings the rate of growth down. When k reaches k*, it remains constant. This means that per capita output, y=f(k), remains constant. Thus, in this version of the model, there is no long-run growth of per capita income.

Notice that a change in the saving rate has no effect on long-run growth rate. It only changes the long-run level of per capita income. (The parameter s has only a level effect and not a growth effect.) This is in contrast to the Harrod-Domar model! What makes the difference? The Solow model assumes that there are diminishing returns to capital, whereas in the Harrod-Domar model there are no diminishing returns. Visually, the smaller the degree of diminishing returns, the closer the sf(k) curve to a straight line. When sf(k) is a straight line, there is no steady state k*. In this case, per capita capital stock can grow indefinitely.

Assuming that the capital-output ratio is constant, the Harrod-Domar model essentially rules out diminishing returns.

With the help of the equation below, we can see that an increase in s leads to an increase in the steady-state level of income, whereas the opposite is true for n and δ.

$$\frac{k^*}{f(k^*)} = \frac{s}{(n+\delta)}$$

Note that a change in population growth rate has a *growth* effect as well as a *level* effect. A higher rate of population growth lowers the steady-state level of per capita income. We know that once steady-state is reached, per capita income will stay constant. With a higher n, total income should grow faster to keep per capita income constant.

Technical/ technological progress:

The Solow model tells us that accumulating capital will not bring growth in the long-run, if the production technology remains the same. What if there is technological progress over time? What if we manage to increase productivity?

This means that the sf(k) curve moves upward over time. You can see that this change will increase the steady-state level of per capita income at the rate of technical progress.

More formally we can show this as follows: Define effective population as $L_t = E_t P_t$, where E_t is the efficiency or productivity of an individual and P_t is the working population, both at time t. Over time, with technical progress, E_t increases. At steady-state, capital per effective labor, K/L, will remain constant, however, K/P will grow at the same rate as E_t.

The equation that governs the path of per capita capital stock is now modified as follows:,

$$(1+\pi)(1+n)\hat{k}_{t+1} = (1-\delta)\hat{k}_t + sf(\hat{k}_t)\,,$$

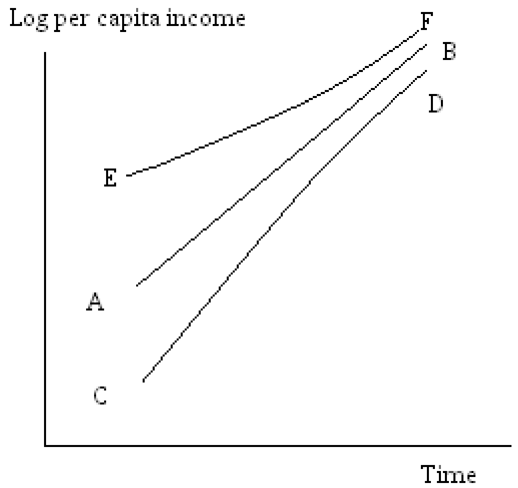
where π is the rate of technical progress and k-hat is the capital stock per efficient population.

(See lecture notes for the continuous–time version of the model.)

Convergence?

1) Unconditional convergence:

Suppose we assume that in the long-run all countries have similar rates of technical progress, saving, population growth and capital depreciation. In such a case, the Solow model predicts that in all countries capital per effective labor converges to a common value, irrespective of the initial conditions in these countries. The unconditional convergence hypothesis requires not only that countries converge to their own steady-states, but that these steady-states are all the same.



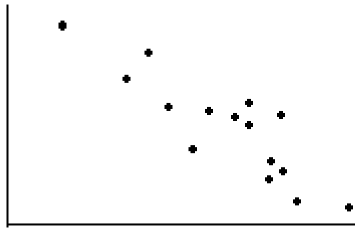
In the figure above, the logarithms of per capita incomes are plotted against time. The slopes indicate the growth rate in per capita income. With constant growth rate in per capita income, the logarithm of per capita income plotted against time appears as a straight line. The line AB shows the growth path of steady-state per capita income. The line CD belongs to a country that starts below the steady-state level. Initially, this country will grow fast. Over time its growth rate will converge to the growth rate implied by the AB line.
A country that starts above the steady-state level of per capita income will grow at a slower rate than the steady-state growth rate. Therefore, unconditional convergence indicates a strong negative relationship between growth rates and initial values of per capita incomes.

To test this hypothesis, we need data. There are two choices: cover a small number of countries over a long period of time, or cover a large number of countries over a short period of time. Both analyses fail to find support for the unconditional convergence hypothesis.

A small number of countries over a long period of time:

Baumol (1986, AER).
Data on per capita income in 1870 from sixteen countries. These countries, sorted from poorest to richest in 1870, are: Japan, Finland, Sweden, Norway, Germany, Italy, Austria, France, Canada, Denmark, the United States, the Netherlands, Switzerland, Belgium, the United Kingdom, and Australia.

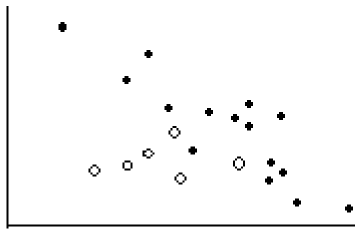
The idea is to plot the data on per capita income in 1870 on the horizontal axis and the growth rate of per capita income over 1870-1979 (measured by the difference in logs) on the vertical axis. If the hypothesis is correct, all observations should approximately lie on a negatively sloped line.



The countries in the dataset had comparable incomes in 1979, but very different incomes in 1870. The problem with this study is that the countries are all rich ex post. Only the countries that have been successful were selected to study convergence.

As an illustration, look at today's successful basketball stars. They came from various backgrounds but they converged to success. However, you cannot say, by looking at them, that a randomly chosen sample of children who aspire to be basketball stars will all succeed!

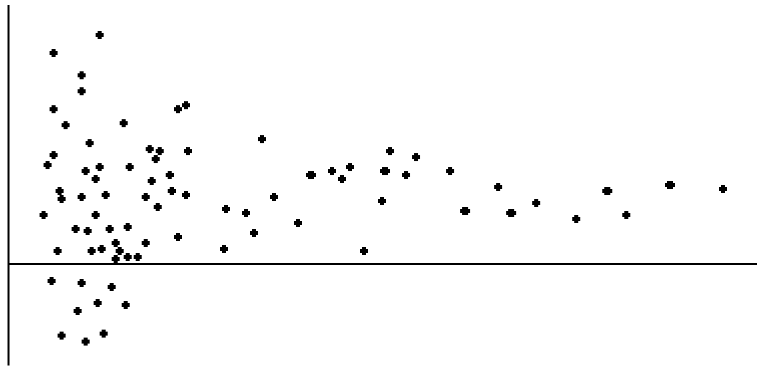
A true test of converge should look at a set of countries that in the past seemed likely to converge to the per capita income of rich countries today. De Long (1988, AER) added to the dataset seven other countries, which appeared as likely to belong to the convergence club as others back in 1870. These were Argentina, Chile, East Germany, Ireland, New Zealand, Portugal, and Spain. With the new dataset, the convergence hypothesis is rejected.



A large number of countries over a short period of time:

Barro (1991, QJE).
Here, the analysis is based on the Summers-Heston dataset over the period 1960-85. Although poor countries have certainly grown, the disparity in relative incomes between them and rich countries has stayed the same, because the poorest countries have grown at about the same rate as the richest.

Plot the average per capita GDP growth rate between 1960-85 on the vertical axis and per capita GDP in 1960 on the horizontal axis. No apparent pattern is visible in the data. Barro finds that the correlation between the two series is 0.09, very close to zero.



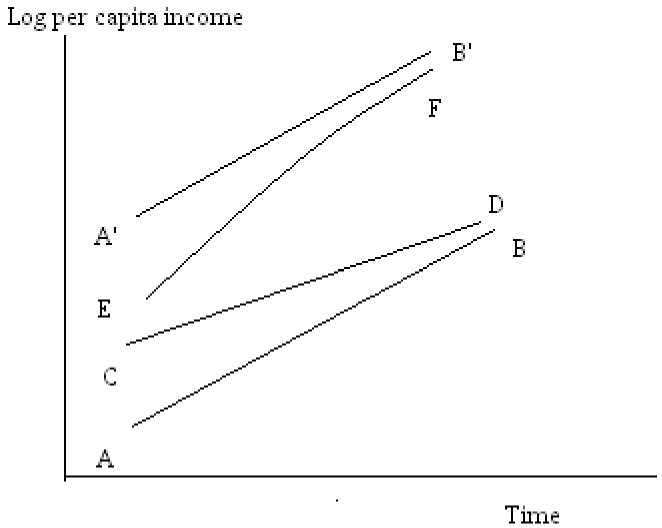
(Remember also that in the mobility matrix analysis we found that there was only a little tendency for countries to move toward a common cluster.)

2) Conditional convergence:

The assumption that saving, population growth and depreciation rates are the same across countries is hard to support. Let's relax this assumption and acknowledge that countries differ in these parameters.

We keep the assumption that knowledge flows freely across countries, so that technological know-how is the same for all countries.

Since in the Solow model the growth rate of per capita income is determined by the rate of technical progress, and since we have assumed this to be the same for all countries, we predict convergence in growth rates. Therefore, conditional convergence hypothesis says that although long-run per capita incomes may vary across countries, long-run per capita growth rates are the same across countries. Countries converge to their own steady-states.



Lines AB and A'B' are parallel. A country with steady-state AB, starting from C, will move along the CD path. Remember that the unconditional convergence hypothesis implied that poor countries grew faster. The conditional convergence hypothesis does not have such a claim. The country that starts from point C is poorer than a country that starts from point E, but it nevertheless grows slower. Growth rate is determined by the position of a country relative to its own steady-state.

How can we test this empirically?

Mankiw, Romer and Weil (1992, QJE).

We now need to control for the saving, population growth and depreciation rates of countries. Remember that with technical progress, we had

$$(1+\pi)(1+n)\hat{k}_{t+1} = (1-\delta)\hat{k}_t + sf(\hat{k}_t).$$

At steady state, $\hat{k}_{t+1} = \hat{k}_t = \hat{k}*$.

Then, $\frac{\hat{k}*}{\hat{y}*} = \frac{s}{(1+n)(1+\pi)-(1-\delta)}$, or approximately $\frac{\hat{k}*}{\hat{y}*} \cong \frac{s}{n+\pi+\delta}$.

Assume that the production function takes the familiar Cobb-Douglas form.

$$y = f(k) = k^{\alpha}.$$

After a few steps of algebra, we get

$$\ln y_t \cong A + \frac{\alpha}{1-\alpha}\ln s - \frac{\alpha}{1-\alpha}\ln(n+\pi+\delta)$$, where A is a constant.

Mankiw, Romer and Weil (1992) took $\pi+\delta$ to be about 5% per year. Using the Summers-Heston data from 1960-85 period, and taking y_t as the 1985 per capita income, they found that more than half the worldwide variation in per capita GDP in 1985 is explained by the two variables s and n. This is a powerful finding. The coefficient estimates were positive and statistically significant for $\ln s$ and negative and statistically significant for $\ln(n+\pi+\delta)$, as predicted by the Solow model. Therefore, we conclude that the Solow model predicts broad relationships that do show up in the data.

Despite its qualitative success, the regression fails to satisfy us quantitatively. Remember that α is the coefficient on capital in the production function. A rough estimate for α is the share of capital in national income, which is around 1/3 in the U.S. This means that $\alpha/1-\alpha$ should be around 1/2. The estimated coefficient on saving is 1.42 and that on population is -1.97. We would expect these to be equal in magnitude and close to ½ in absolute value.

To conclude, we have found that if we relax the assumption that the saving and population growth rates are the same across countries (and allow them to be country-specific) then the Solow model makes some sense of the data. However, this leaves the question of why these rates are different unanswered. If all human beings are driven by the same economic motivations and there are no fundamental genetic differences in desires to save, then the differences that we observe may be caused by their particular economic experiences (i.e. these parameters are endogenous), rather than by some irreconcilable cultural or social differences. A fuller theory of growth should account for why saving and population growth rates are different.

We have found that although variations in saving and population growth rates can explain the direction of variations in per capita income, the observed differences are much larger than what is predicted by theory. This suggests that there is little or no diminishing returns to capital, which is hard to buy. We need new theories that will resolve this conflict.

Another criticism to the model is about the assumptions that the rate of technical progress is exogenous and technical progress flows freely among countries. If all long-run growth comes from technical progress, then we need to understand what forces shape technical progress and its dissipation.