QUIZ - 9

Question 1.

$\bar{X} = 150$

$n = 16$

$s = 12$

a)

assumptions:

- population standard deviation is unknown
- population is normally distributed.

$$\bar{X} - t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

$1-\alpha = 0.95$

$\alpha = 0.05 \Rightarrow \alpha/2 = 0.025$

$n-1 = 15$

$$\left.\right\} t_{15,0.025} = 2.131$$

$$\bar{X} \mp t_{15,0.025}\frac{s}{\sqrt{n}} = 150 \mp 2.131 \cdot \frac{12}{\sqrt{16}} = 150 \mp 6.393$$

The confidence interval is:

$$143.607 < \mu < 156.393$$

b) $1-\alpha = 0.99$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$t_{15,\ 0.005} = 2,947$

$$\bar{X} \mp t_{n-1,\alpha/2}\frac{s}{\sqrt{n}} = 150 \mp (2,947) \cdot \frac{12}{\sqrt{16}} = 150 \mp 8.841$$

The confidence interval is $141,159 < \mu < 158.841$ and wider.

Question 2:

a) $n = 400$

$\hat{p} = \frac{320}{400} = 0.8 \qquad 1 - \hat{p} = 0.2$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < P < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\left.\begin{array}{l} 1-\alpha = 0.90 \\ \alpha = 0.10 \\ \alpha/2 = 0.05 \end{array}\right\} z_{0.05} = 1.645 \quad \text{(from the table)}$$

$$\hat{p} \mp z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.8 \mp (1.645)\cdot\sqrt{\frac{0.8 \cdot 0.2}{400}} = 0.8 \mp (0.0329)$$

The confidence interval is

$0.7671 < p < 0.8329$

b) The width of the interval that estimated in part (a)

$$2 \cdot z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 2 \cdot (1.645)(0.02) = 0.0658$$