İKT 253 Statistics June 13, 2016

Quiz 6 CEVAPLAR

Student's Full Name and Number:

(2) Question 1: A customer service center in India receives, on average, 4.2 phone calls per minute. If the distribution of calls is Poisson, what is the probability of receiving at least three calls in a minute? $\lambda = 4{,}2$ $\quad P(X \geq 3) = 1 - P(X<3) = 1 - [P(2)+P(1)+P(0)]$

$$P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$= 1 - e^{-4{,}2}(14{,}2)$$

$$P(2)+P(1)+P(0) = e^{-4{,}2}\left(\frac{(4{,}2)^2}{2!} + \frac{(4{,}2)^1}{1!} + \frac{(4{,}2)^0}{0!}\right) = e^{-4{,}2}(14{,}2)$$

Question 2: Consider the joint probability distribution of X and Y:

		X	
		1	2
Y	0	0.30	0.20
	1	0.25	0.25

xy	P(x,y)
0	0,50
1	0,25
2	0,25

(2) a) Compute the marginal probability distributions for X and Y.

$P(x=1) = \sum_y (1,y) = 0{,}30 + 0{,}25 = 0{,}55$ | $P(X=2) = \sum_y P(2,y) = 0{,}20 + 0{,}25 = 0{,}45$

$P(y=0) = \sum_x (x,0) = 0{,}30 + 0{,}20 = 0{,}50$ | $P(y=1) = \sum_x P(x,1) = 0{,}25 + 0{,}25 = 0{,}50$

(4) b) Compute the covariance and correlation for X and Y.

$$\text{cov}(X,Y) = \sum_x \sum_y xy\,P(x,y) - \mu_x \mu_y = 1\cdot(0{,}25) + 2\cdot(0{,}25) - (1{,}45)(0{,}50) = \underline{0{,}025}$$

$$\mu_x = E(x) = \sum_x x \cdot P_x = 1\cdot(0{,}55) + 2\cdot(0{,}45) = 1{,}45$$

$$\mu_y = E(y) = \sum_y y \cdot P_y = 0\cdot(0{,}50) + 1\cdot(0{,}50) = 0{,}50$$

Not: Corr(x,y) çözümü diğer sayfada.

(2) c) Compute the mean and variance for W = 2X + Y.

$$\text{Mean of } W = 2\mu_x + \mu_y = 2(1{,}45) + 0{,}50 = 3{,}40$$

$$\text{Variance of } W = 4 \cdot \text{Var}X + \text{Var}Y + 2\cdot 2\,\text{Cov}(x,y)$$

$$= 4 \cdot \sigma_x^2 + \sigma_y^2 + 4\cdot(0{,}025)$$

$$= 4\cdot(0{,}2475) + 0{,}25 + 0{,}1$$

$$= \underline{1.34}$$

Question 2:

b) $\mathrm{corr}(x,y) = \dfrac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y}$

$$\sigma_x^2 = \sum_x (x-\mu_x)^2 P(x) = \sum_x x^2 P_x - \mu_x^2$$

$$= 0.55 \cdot 1 + 0.45 \cdot (2^2) - (1.45)^2$$

$$= 0.55 + 1.8 - 2.1025$$

$$\sigma_x^2 = 0.2475 \Rightarrow \sigma_x \cong 0.497.$$

$$\sigma_y^2 = \sum_y (y-\mu_y)^2 P(y) = \sum_y y^2 P_y - \mu_y^2$$

$$= 0^2 \cdot 0.50 + 1^2 \cdot 0.50 - (0.50)^2$$

$$\sigma_y^2 = 0.25 \Rightarrow \sigma_y = 0.5$$

$$\mathrm{corr}(x,y) = \frac{0.025}{\underbrace{0.497}_{\cong 0.5} \cdot 0.5} = 0.1006\ldots \cong 0.1$$

→ Positive correlation,

If x increases, y increases.