

Quiz 4

Student's Full Name and Number:

Question 1: Fiona owns a used car dealer with three Mercedes (M_1, M_2, M_3) and two Toyotas (T_1, T_2). Two customers arrive; each one is interested in buying one car. The customers do not know each other and they do not communicate. Let A be the event that the customers select at least one Toyota. Let B be the event that the customers select two cars of the same model.

a) Describe the sample space.

$$S = \{M_1T_1, T_1M_1, M_2T_1, T_1M_2, M_3T_1, T_1M_3, M_1T_2, T_2M_1, M_2T_2, T_2M_2, M_3T_2, T_2M_3, M_1M_2, M_2M_1, M_1M_3, M_3M_1, M_2M_3, M_3M_2\}$$

b) Describe event A. What is the probability of A?

$$A = \{M_1T_1, T_1M_1, M_2T_1, T_1M_2, M_3T_1, T_1M_3, M_1T_2, T_2M_1, M_2T_2, T_2M_2, M_3T_2, T_2M_3, T_1T_2, T_2T_1\}$$

c) Describe event B. What is the probability of B?

$$B = \{M_1M_2, M_2M_1, M_1M_3, M_3M_1, M_2M_3, M_3M_2, T_1T_2, T_2T_1\}$$

d) Describe event \bar{A} . What is the probability of \bar{A} ?

$$\bar{A} = \{M_1M_2, M_2M_1, M_1M_3, M_3M_1, M_2M_3, M_3M_2\}$$

Question 2: It is estimated that 30% of all automobile accidents were caused by weather conditions and that 20% of all automobile accidents involved bodily injury. Of those accidents that involved bodily injury, 40% were caused by weather conditions

a) What is the probability that a randomly chosen accident both was caused by weather conditions and involved bodily injury? $P(W \cap B) = ?$

$$P(W) = 0.30 \quad P(W|B) = 0.40 \quad P(W \cap B) = P(W|B) \cdot P(B) \\ P(B) = 0.20 \quad = 0.40 \cdot 0.20 \\ = 0.08$$

b) Are the events "caused by weather conditions" and "involved bodily injury" independent?

$$P(W) \cdot P(B) \stackrel{?}{=} P(W \cap B) \\ 0.06 \neq 0.08, \text{ so they are not independent.}$$

c) If a randomly chosen accident was caused by weather conditions, what is the probability that it involved bodily injury? $P(B|W) = ?$

$$P(B|W) = \frac{P(B \cap W)}{P(W)} = \frac{0.08}{0.30} = \frac{8}{30}$$

d) What is the probability that a randomly chosen accident both was not caused by weather conditions and did not involve bodily injury? $P(\bar{W} \cap \bar{B}) = ?$

$$P(\bar{W} \cap \bar{B}) = P(\overline{W \cup B}) = 1 - P(W \cup B)$$

$$P(W \cup B) = P(W) + P(B) - P(W \cap B) \\ = 0.30 + 0.20 - 0.08 \\ = 0.42$$

$$P(\bar{W} \cap \bar{B}) = 1 - 0.42 = 0.58$$

Question 1

b) $P(A) = ?$

$$P(A) = 1 - P(\text{no Toyota}) = 1 - P(\text{all Mercedes})$$

$$= 1 - \frac{P_2^3}{P_2^5} = 1 - \frac{6}{20} = \underline{\underline{\frac{14}{20}}}$$

c) $P(B) = ?$

$$P(B) = P(\text{two Toyota}) + P(\text{two Mercedes})$$

$$= \frac{P_2^2}{P_2^5} + \frac{P_2^3}{P_2^5} = \frac{2}{20} + \frac{6}{20} = \underline{\underline{\frac{8}{20}}}$$

d) $P(\bar{A}) = ?$

$$P(\bar{A}) = P(\text{no Toyota}) = P(\text{all Mercedes})$$

$$= \frac{P_2^3}{P_2^5} = \underline{\underline{\frac{6}{20}}}$$