

Quiz 2

Student's Full Name and Number:

Question 1: Suppose that you have the following observations for variable X.

- 6 21 29 31 34 35 38 44 52 70
- 7 21 29 31 34 35 38 44 54 72
- 8 22 30 31 34 35 39 45 55 75
- 10 25 30 32 34 35 39 45 58 76
- 11 25 30 32 34 35 39 45 62 80
- 12 26 30 32 34 35 40 46 63 83
- 16 26 30 33 34 35 43 48 63 88
- 19 28 31 33 34 36 44 50 69 88
- 20 29 31 33 34 37 44 51 70 98
- 20 29 31 34 35 37 44 51 70

Fill in the blanks:

- a) 25 percent of the observations take a value of30..... or lower. 50 percent of the observations take a value of35..... or lower. 75 percent of the observations take a value of45..... or lower.
- b) The first quartile (Q_1) is equal to30....., the second quartile (Q_2) is equal to35....., and the third quartile (Q_3) is equal to45...... The interquartile range is equal to15.....
- c) The median is equal to35.....

Question 2: If the mean of a population is 30 and the standard deviation is 6, then at least what proportion of observations are between 21 and 39? At least what proportion of observations are between 18 and 42? (Hint: Use Chebychev's Theorem.)

$21 = 30 - 6k$
 $39 = 30 + 6k$

$12k = 18$
 $k = 3/2$

$\Rightarrow \left[1 - \left(\frac{1}{(3/2)^2}\right)\right] = \frac{5}{9}$
 $\frac{5}{9} \approx 0.55$

$18 = 30 - 6k$
 $42 = 30 + 6k$

$12k = 24$
 $k = 2$

$\Rightarrow 1 - \frac{1}{4} = \frac{3}{4}$
 $\frac{3}{4} \approx 0.75$

Question 3: Let X be the number of candies you eat and Y be the length of exercise needed to burn the calories of the candies. Fill in the blanks in the table and answer the questions below.

x_i (pieces)	y_i (minutes)	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
1	5	-2	4	-10	100	20
2	10	-1	1	-5	25	5
3	15	0	0	0	0	0
4	20	1	1	5	25	5
5	25	2	4	10	100	20

- a) What is the sample variance of X?
- b) What is the sample variance of Y?
- c) What is the sample covariance between X and Y?
- d) What is the sample correlation coefficient between X and Y?

a) Sample variance of $X = S_x^2 = \frac{\sum_{i=1}^5 (x_i - \bar{x})^2}{5-1} = \frac{10}{4} = 2,5$

b) Sample variance of Y

$$S_y^2 = \frac{\sum_{i=1}^5 (y_i - \bar{y})^2}{5-1} = \frac{250}{4} = 62,5$$

c) Sample covariance between X and Y

$$S_{xy} = \frac{\sum_{i=1}^5 (x_i - \bar{x})(y_i - \bar{y})}{5-1} = \frac{50}{4} = 12,5$$

d) Sample correlation coefficient between X and Y

$$r = \frac{\text{cov}(X, Y)}{S_x S_y} = \frac{12,5}{\sqrt{2,5 \cdot 62,5}} = \frac{12,5}{\sqrt{156,25}} = 1$$