**Question 1:** (3+3 = 6 points)

Suppose that a scientist applies for funding to several research foundations. Assume that applications are independent from each other and each application has a 25% probability of success. The scientist makes 10 applications.

a) What is the probability that out of 10 applications exactly 3 are successful?

Here, the number of successes (X) has a binomial distribution.

$$P(X=3) = \binom{10}{3}(0.25)^3(0.75)^7 = \frac{10!}{3!7!}(0.25)^3(0.75)^7$$

b) What is the probability that at least one application is successful?

$$P(\text{at least 1 appl. is successful}) = 1 - P(\text{no appl. is successful})$$

$$= 1 - P(X=0) = 1 - \binom{10}{0}(0.25)^0(0.75)^{10} = 1-(0.75)^{10}$$

**Question 2:** (3+3 = 6 points)

Suppose that a random variable X takes the following four values: -2, -1, 0, and 2.

P(-2) = 0.30, P(-1) = 0.20, P(0) = 0.10, P(2) = 0.40.

a) What is the expected value of X?

$$E(X) = \sum_x x \cdot P(x) = (-2)(0.30) + (-1)(0.20) + 0(0.10) + 2(0.40)$$

$$= (-0.60) + (-0.20) + 0 + 0.80 = 0 = \mu_X$$

b) What is the variance of X?

$$Var(X) = E[(X-\mu_X)^2] = \sum_x (X-\mu_X)^2 P(x)$$

$$= (-2-0)^2(0.30) + (-1-0)^2(0.20) + (0-0)^2(0.10) + (2-0)^2(0.40)$$

$$= 4(0.30) + 1(0.20) + 0 + 4(0.40)$$

$$= 1.20 + 0.20 + 1.60 = 3$$

**Question 3:** (3+3 = 6 points)

A bookstore owner has observed that 30% of customers are university students, 20% of customers are high school students, and the remaining are teachers. She has also observed that 60% of university students, 20% of high school students, and 80% of teachers who visit the store make a purchase.

a) What is the probability that a randomly chosen customer visiting the store will make a purchase?

$P(U) = 0.30, \quad P(HS) = 0.20, \quad P(T) = 0.50$

$$P(\text{purchase}) = P(\text{Purchase}|U).P(U) + P(\text{Purchase}|HS).P(HS) + P(\text{Purchase}|T).P(T)$$

$$= (0.60)(0.30) + (0.20)(0.20) + (0.80)(0.50)$$

$$= 0.18 + 0.04 + 0.40 = 0.62 = 62\%$$

b) If a randomly chosen customer makes a purchase, what is the probability that the customer is a university student?

$$P(U|\text{Purchase}) = \frac{P(U \cap \text{Purchase})}{P(\text{Purchase})} = \frac{P(\text{Pur}|U).P(U)}{P(\text{Purchase})}$$

$$= \frac{0.18}{0.62} = \frac{18}{62} = \frac{9}{31}$$

**Question 4:** (2+2+2= 6 points)

There are 8 coaches and 12 students who wish to be members of the athletic council. The council has 5 members. Mr. Smith, the school principal, will select them. How many different councils can be formed in each case below?

a) There are no restrictions on the number of students and coaches to be selected

$$\binom{20}{5} = \frac{20!}{15!\,5!}$$

b) There must be exactly 3 coaches and 2 students in the athletic council

$$\binom{8}{3}\binom{12}{2} = \frac{8!}{3!\,5!} \cdot \frac{12!}{10!\,2!}$$

c) There must be at least one coach in the athletic council

All possible combinations − the combinations with no coach

$$\binom{20}{5} - \binom{8}{0}\binom{12}{5}$$

OR

$$\binom{8}{1}\binom{12}{4} + \binom{8}{2}\binom{12}{3} + \binom{8}{3}\binom{12}{2} + \binom{8}{4}\binom{12}{1} + \binom{8}{5}\binom{12}{0}$$

**Question 5:** (1+1+1+2+1=6 points)

Exam grades of 5 students are given below:

60, 70, 75, 50, 55

Please find:

a. The mean

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{60+70+75+50+55}{5} = 62$$

b. The mode

No mode.

c. The median

50 55 (60) 70 75 Median = 60

d. The variance

$$s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \frac{(60-62)^2 + (70-62)^2 + (75-62)^2 + (50-62)^2 + (55-62)^2}{4}$$

$$= \frac{4 + 64 + 169 + 144 + 49}{4}$$

$$s^2 = 107.5$$

e. The standard deviation

$$s = \sqrt{s^2} = \sqrt{107.5}$$

$$= 10.37$$