9.44 Of a random sample of 199 auditors, 104 indicated some measure of agreement with this statement: "Cash flow is an important indication of profitability."

a. Test at the 10% significance level against a two-sided alternative the null hypothesis that one-half of the members of this population would agree with this statement. Also find and interpret the p-value of this test.

b. Find the probability of accepting the null hypothesis with a 10%-level test if, in fact, 60% of all auditors agree that cash flow is an important indicator of profitability.

a. H_0 is not rejected when $-1.645 > \frac{p-.5}{\sqrt{.25/199}} > 1.645$ or when .442 > p > .558. Since the sample proportion is .5226, it is within the critical values. The decision is that there is insufficient evidence to reject the null hypothesis.

b. $\beta = P(\frac{.442-.6}{\sqrt{(.6)(.4)/199}} < Z < \frac{.558-.6}{\sqrt{(.6)(.4)/199}})$. Power= 1-P(-4.55 < Z < -1.21) = .1131

9.39

A company that receives shipments of batteries tests a random sample of nine of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with a standard deviation of 3 hours. For one particular shipment the mean lifetime for a sample of nine batteries was 48.2 hours.

a. Test at the 10% level the null hypothesis that the population mean lifetime is at least 50 hours.

b. Find the power of a 10%-level test when the true mean lifetime of batteries is 49 hours.

a. H_0 is rejected when $\frac{\bar{X}-50}{3/\sqrt{9}} < -1.28$ or when $\bar{X} < 48.72$. Given an $\bar{X} = 48.2$ hours, the decision is to reject the null hypothesis.

b. The power of the test = 1 - β = 1 – P(Z > $\frac{48.72-49}{3/\sqrt{9}}$) = 1 – P(Z > -.28) = .3897

9.37 Consider a problem with the hypothesis test

$$H_0 : \mu = 5$$
$$H_1 : \mu > 5$$

and the following decision rule:

$$\text{Reject } H_0 \text{ if } \frac{\bar{x} - 5}{0.1/\sqrt{16}} > 1.645 \quad \text{or}$$

$$\bar{x} > 5 + 1.645(0.1/\sqrt{16}) = 5.041$$

Compute the probability of Type II error and the power for the following true population means:

a. $\mu = 5.10$
b. $\mu = 5.03$
c. $\mu = 5.15$
d. $\mu = 5.07$

Compute the probability of Type II error and the power for the following

a. $\mu = 5.10$. $\beta = P(\bar{x} \le \bar{x}_c \mid \mu = \mu^*)$ = $P(\bar{x} \le 5.041 \mid \mu^* = 5.10)$ = $P\left(z \le \frac{5.041-5.10}{.1/\sqrt{16}}\right)$

= P(z ≤ -2.36) = .0091. Power = 1 – .0091 = .9909

b. $\mu = 5.03$. $\beta = P(\bar{x} \le \bar{x}_c \mid \mu = \mu^*)$ = $P(\bar{x} \le 5.041 \mid \mu^* = 5.03)$ = $P\left(z \le \frac{5.041-5.03}{.1/\sqrt{16}}\right)$

= P(z ≤ .44) = .6700. Power = 1 – .6700 = .3300

c. $\mu = 5.15$. $\beta = P(\bar{x} \le \bar{x}_c \mid \mu = \mu^*)$ = $P(\bar{x} \le 5.041 \mid \mu^* = 5.15)$ = $P\left(z \le \frac{5.041-5.15}{.1/\sqrt{16}}\right)$

= P(z ≤ -4.36) = .0000. Power = 1 – .0000 = 1.0000

b. $\mu = 5.07$. $\beta = P(\bar{x} \le \bar{x}_c \mid \mu = \mu^*)$ = $P(\bar{x} \le 5.041 \mid \mu^* = 5.07)$ = $P\left(z \le \frac{5.041-5.07}{.1/\sqrt{16}}\right)$

= P(z ≤ -1.16) = .3770. Power = 1 – .3770 = .6230

9.38 Consider Example 9.6 with the null hypothesis

$$H_0 : P = P_0 = 0.50$$

and the alternative hypothesis

$$H_0 : P \neq 0.50$$

The decision rule is

$$\frac{\hat{p}_x - 0.50}{\sqrt{0.50(1 - 0.50)/600}} < -1.96 \quad \text{or}$$

$$\frac{\hat{p}_x - 0.50}{\sqrt{0.50(1 - 0.50)/600}} > 1.96$$

with a sample size of $n = 600$. What is the probability of Type II error if the actual population proportion is

a. $P = 0.52$?
b. $P = 0.58$?
c. $P = 0.53$?
d. $P = 0.48$?
e. $P = 0.43$?

What is the probability of Type II error if the actual proportion is

a. $P = .52$. $\beta = P(.46 \le \hat{p} \le .54 \mid p = p^*)$ = $P\left[\frac{.46 - p^*}{\sqrt{\frac{p^*(1-p^*)}{n}}} \le z \le \frac{.54 - p^*}{\sqrt{\frac{p^*(1-p^*)}{n}}}\right]$

= $P\left[\frac{.46 - .52}{\sqrt{\frac{.52(1-.52)}{600}}} \le z \le \frac{.54 - .52}{\sqrt{\frac{.52(1-.52)}{600}}}\right]$ = P(-2.94 ≤ z ≤ .98) = .4984 + .3365 = .8349

b. $P = .58$. $\beta = P(.46 \le \hat{p} \le .54 \mid p = p^*)$ = $P\left[\frac{.46 - .58}{\sqrt{\frac{.58(1-.58)}{600}}} \le z \le \frac{.54 - .58}{\sqrt{\frac{.58(1-.58)}{600}}}\right]$

= P(-5.96 ≤ z ≤ -1.99) = .5000 – .4767 = .0233

c. $P = .53$. $\beta = P(.46 \le \hat{p} \le .54 \mid p = p^*)$ = $P\left[\frac{.46 - .53}{\sqrt{\frac{.53(1-.53)}{600}}} \le z \le \frac{.54 - .53}{\sqrt{\frac{.53(1-.53)}{600}}}\right]$

= P(-3.44 ≤ z ≤ .49) = .4997 + .1879 = .6876

d. $P = .48$. $\beta = P(.46 \le \hat{p} \le .54 \mid p = p^*)$ = $P\left[\frac{.46 - .48}{\sqrt{\frac{.48(1-.48)}{600}}} \le z \le \frac{.54 - .48}{\sqrt{\frac{.48(1-.48)}{600}}}\right]$

= P(-.98 ≤ z ≤ 2.94) = .3365 + .4984 = .8349

e. $P = .43$. $\beta = P(.46 \le \hat{p} \le .54 \mid p = p^*)$ = $P\left[\frac{.46 - .43}{\sqrt{\frac{.43(1-.43)}{600}}} \le z \le \frac{.54 - .43}{\sqrt{\frac{.43(1-.43)}{600}}}\right]$

= P(1.48 ≤ z ≤5.44) = .5000 – .4306 = .0694