ECON 551 Quantitative Methods- Part 1: Probability and Statistics

<u>Homework 4</u>

1. Suppose that the r.v.s X_1 and X_2 are independent with normal distributions $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$. Let $= a + bX_1 + cX_2$. Derive the pdf of U.

2. Let $\bar{X}$ be the sample mean of a random sample of size 5 from a normal distribution with mean =0 and variance = 125. Determine c such that $P(\bar{X} < c) = 0.90$.

3. If $\bar{X}$ is the sample mean of a random sample of size n from a normal distribution with mean $= \mu$ and variance = 100. Determine n so that $P(\mu - 5 < \bar{X} < \mu + 5) = 0.954$.

4. Suppose $X_1, X_2, \ldots, X_{25}$ and $Y_1, Y_2, \ldots, Y_{25}$ are two independent random samples from two normal distributions $N(0,16)$ and $N(1,9)$, respectively. Let $\bar{X}$ and $\bar{Y}$ denote the corresponding sample means. Compute $P(\bar{X} > \bar{Y})$.

5. Find the mean and variance of $S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/n$, where $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$.

6. Let S^2 be the sample variance of a random sample of size 6 from the normal distribution $N(\mu, 12)$. Find $P(2.3 < S^2 < 22.2)$.

7. Let $\bar{X}$ and S^2 be the sample mean and sample variance of a random sample of size 25 from a distribution that is $N(3, 100)$. Evaluate $P(0 < \bar{X} < 6 \ and \ 55.2 < S^2 < 145.6)$.

8. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size 4 from the distribution having pdf $f_X(x) = e^{-x}, 0 < x < \infty$, and zero elsewhere. Find $P(3 < Y_4)$.

9. Let Xand Y be random variables with means μ_1, μ_2; variances σ_1^2 , σ_2^2; and correlation coefficient ρ. Show that the correlation coefficient of $W = aX + b, a > 0$, and $Z = cY + d, c > 0$ is ρ.

10. What is the probability that at least one observation of a random sample of size $n = 5$ from a continuous-type distribution exceeds the 90th percentile? (We do not know what the pdf is, but we know what the cumulative distribution function is.)

11. The joint pdf of Xand Y is defined as $f_{X,Y}(x, y) = 1/2$. This pdf is defined over the rectangle whose four corners are (1,0), (0,1), (-1,0), (0,-1). What is the marginal density $f_Y(y)$?