ECON 551 Quantitative Methods- Part 1: Probability and Statistics

Homework 3

1. The r.v. X has the pdf $f_X(x) = 1/3\,, -1 < x < 2$ and zero elsewhere. Show that the mgf is: $M(t) = (e^{2t} - e^{-t})/3t\,,\ \ t \neq 0,$ and $M(0) = 1.$

2. Let $\mu = E(X)$, and $\sigma = \sqrt{E(X-\mu)^2}$.

For the following pdf's, compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$.

a) $f(x) = 6x(1-x),\ 0 < x < 1;\ \ zero\ elsewhere.$

b) $f(x) = 2^{-x}\ x = 1, 2, 3, \ldots;\ \ zero\ elsewhere.$

In part (b) keep in mind that $\sum_{x=0}^{\infty} a^x = \frac{1-a^x}{1-a}, 0 < a < 1;\quad \sum_{x=0}^{\infty} x\, a^{x-1} = \frac{1}{(1-a)^2}.$

You may also need $E(X^2) = 2E(X^2) - E(X^2)$.

3. Let the r.v.'s X and Y have the joint pdf
$f_{X,Y}(x,y) = \frac{1}{3},\ \ if(x,y) = (0,0), (1,1), (2,0);\ \ zero\ elsewhere.$

We know that when two r.v.'s are independent, the correlation coefficient between them is zero. Show that a zero correlation coefficient does not imply that the two r.v.'s are independent.

4. Suppose that the r.v. $X \sim N(0,1)$. Let $= a + bX$. Derive the pdf of U.

5. Suppose that the r.v. $X \sim N(\mu, \sigma^2)$. Let $= (Y - \mu)/\sigma$. Derive the pdf of U.

6. Let the continuous r.v. X have the pdf whose graph is symmetric around $x = c$. Show that if the mean of this distribution exists, it is equal to c.

7. If the r.v. X has the Poisson distribution such that $f(1) = f(2)$, find $f(4)$.

8. The mgf of a r.v. X is known to be $\exp(4(e^t - 1))$, where exp (.) is the exponential function. Show that $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.93$.

9. In a lengthy manuscript, it is discovered that only 13.5 percent of the pages contain no error. If we assume that the number of errors per page is a r.v. with Poisson distribution, find the percentage of pages with exactly one error.

10. Let ϕ and Φ be the pdf and distribution function, respectively, of a r.v. with a $N(0,1)$ distribution. Let Y have a truncated distribution with pdf $\phi(y)/[\Phi(\mathrm{b}) - \Phi(\mathrm{a})]\,, a < y < b$. Show that $E(Y)$ is equal to $[\phi(a) - \phi(b)]/[\Phi(\mathrm{b}) - \Phi(\mathrm{a})]$.

11. If the mgf of a r.v. X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $P(X = 2\ or\ 3)$.

12. Let Y be the number of successes in n independent repetitions of a random experiment having a probability of success $p = 1/4$. Determine the smallest value of n so that $P(Y \geq 1) \geq 0.70$. (Think of this situation. Suppose your probability of passing an exam is ¼. Then at least how many times should you take this exam so that you will pass the exam at least once with at least 70% probability?)