

## ECON 551 Quantitative Methods- Part 1: Probability and Statistics

### Homework 2

1. In a class of 20 students, what is the probability that exactly two students have the same birthday?
2. Find the mode for each of the following distributions. (Which value of  $X$  maximizes the pdf?)
  - a)  $f(x) = (1/2)^x$ ,  $x = 1, 2, 3, \dots$ , zero elsewhere.
  - b)  $f(x) = 12x^2(1 - x)$ ,  $0 < x < 1$ , zero elsewhere.
  - c)  $f(x) = (1/2)x^2e^{-x}$ ,  $0 < x < \infty$ , zero elsewhere.
3. Find the median of each of the following distributions:
  - a)  $f(x) = 3x^2$ ,  $0 < x < 1$ , zero elsewhere.
  - b)  $f(x) = 1/(\pi(1 + x^2))$ ,  $-\infty < x < \infty$ .
4. Find the distribution function  $F(x)$  associated with each of the following probability density functions. Sketch the graphs of  $f(x)$  and  $F(x)$ .
  - a)  $f(x) = 3(1 - x)^2$ ,  $0 < x < 1$ , zero elsewhere.
  - b)  $f(x) = 1/x^2$ ,  $1 < x < \infty$ , zero elsewhere.
  - c)  $f(x) = 1/3$ ,  $0 < x < 1$  or  $2 < x < 4$ , zero elsewhere.
5. Four distinct integers are chosen at random and without replacement from the first 10 positive integers  $(1, 2, \dots, 10)$ . Let the random variable  $X$  be the next to smallest of these four numbers. Find the p.d.f. of  $X$ .
6. Let  $X$  be a r.v. of the continuous type with pdf  $f(x)$ , which is positive provided that  $0 < x < b < \infty$ , and is equal to zero otherwise. Show that  $E(X) = \int_0^b [1 - F(x)] dx$ , where  $F(x)$  is the distribution function of  $X$ .
7. Let the r.v.  $X$  have mean  $\mu$ , standard deviation  $\sigma$ . Show that  $E\left(\frac{X-\mu}{\sigma}\right) = 0$  and that  $E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] = 1$ .
8. The r.v.  $X$  has the pdf  $f(x) = \frac{1}{x^2}$ ,  $1 < x < \infty$  and zero elsewhere. Find  $E(X)$ .
9. Let  $f_{X,Y}(x,y) = e^{-x-y}$ ,  $0 < x < \infty$ ,  $0 < y < \infty$ . Let's define  $Z = X + Y$ . What are the (cumulative) distribution function and the probability density function of  $Z$ ?
10. We draw 13 cards from a deck of 52 cards, at random and without replacement. Let  $X$  be the number of spades in these 13 cards and  $Y$  be the number of hearts in these 13 cards. What are the pdf's of  $X$  and  $Y$ ? What is the joint pdf of  $X$  and  $Y$ ?
11. Let  $X$  and  $Y$  have the joint pdf  $f_{X,Y}(x,y) = 15x^2y$ ,  $0 < x < y < 1$ , and zero elsewhere.
  - a) Show that the joint pdf sums to one.
  - b) Find  $f_X(x)$ , the marginal pdf of  $X$ .
  - c) Find  $f_Y(y)$ , the marginal pdf of  $Y$ .
  - d) Find  $P(X + Y) \leq 1$ .