

ECON 551 Quantitative Methods- Part 1: Probability and Statistics

Homework 1

1. A random experiment consists of drawing cards from an ordinary deck of 52 cards. There is an equal probability of drawing any card. Suppose 13 cards are drawn from this deck at random and without replacement. Let A denote the collection of 13 hearts and B the collection of 4 kings. Compute $P(A)$, $P(B)$, $P(A \cap B)$ and $P(A \cup B)$.

2. If the sample space is $\Omega = A \cup B$ and if $P(A)=0.8$, $P(B)=0.5$, find $P(A \cap B)$.

3. If A and B are subsets of the sample space Ω , show that

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$

4. If A , B and C are subsets of Ω , show that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

5. Three distinct integers are drawn at random from the first 20 positive integers. Compute the probability that

a) their sum is even;

b) their product is even.

6. A hand of 13 cards is to be drawn at random and without replacement from a deck of playing cards. Find the conditional probability that there are at least three kings in the hand given that the hand contains at least two kings.

7. If A and B are independent events, show that the following pairs of events are also independent:

a) A and B^c ;

b) A^c and B ;

c) A^c and B^c .

8. A certain disease is present in about 1 out of 1000 persons in a given population. A testing program is to be carried out using a detection device that gives a positive reading with probability 0.99 for a diseased person and with probability 0.05 for a healthy person.

a) What is the probability that a person who has a positive reading actually has the disease?

b) How does your answer to (a) change if the frequency of the disease is 1 in 100 or 1 in 10?

9. In a class of 30 students, what is the probability that at least two students have the same birthday?