İKT253 HW4

① $n=61$, $\bar{X}=2.42$, $s=1$

ⓐ $\alpha=0.05$, two sided test

$H_0: \mu=2.0$

$H_1: \mu \neq 2.0$

$$\boxed{\frac{\bar{X}-\mu_0}{s/\sqrt{n}} \rightarrow t \text{ distribution}}$$

reject H_0 if
- $\bar{X} < \mu_0 - t_{n-1,\alpha/2}\, s/\sqrt{n}$
- $\bar{X} > \mu_0 + t_{n-1,\alpha/2}\, s/\sqrt{n}$

- $\bar{X} < 2 - t_{60,\,0.025}\, 1/\sqrt{61}$

  $\bar{X} < 2 - 2\times 1/\sqrt{61}$

  $\bar{X} < 1.743$

- $\bar{X} > 2 + t_{60,\,0.025}\, 1/\sqrt{61}$

  $\bar{X} > 2 + 2\times 1/\sqrt{61}$

  $\bar{X} > 2.256$

$\bar{X} = 2.42 > 2.256$

Therefore, we reject H_0

ⓑ for the size of p-value,

$$\frac{\bar{X}-\mu_0}{s/\sqrt{n}} = \frac{2.42-2}{1/\sqrt{61}} = 3.280$$

$P(t_{60} > 3.280) < P(t_{60} > 2.660)$ → the last value that I got from t-table when $n-1=60$

$= P(t_{60} > 3.280) < 0.005$

p-value $= 2.P(t_{60} > 3.280) < 2.(0.005)$

p-value $= 2.P(t_{60} > 3.280) < 0.010$

p-value $< \alpha = 0.05$

reject H_0

If p-value $< \alpha$
reject H_0
If p-value $\geq \alpha$
do not reject H_0

① ⓒ Type II error probability and power?

$\mu = 2.047$

$\beta = P(\text{Do not reject } H_0 \mid \mu = 2.047)$

$= P(1.743 < \bar{X} < 2.256 \mid \mu = 2.047)$

$$= P\left(\frac{1.743 - 2.047}{1/\sqrt{61}} < \frac{\bar{X} - 2.047}{1/\sqrt{61}} < \frac{2.256 - 2.047}{1/\sqrt{61}}\right)$$

$= P(-2.374 < t_{60} < 1.632)$

$\Rightarrow F(1.632) - F(-2.374)$

$= F(1.632) - (1 - F(2.374))$

$\Rightarrow 0.95 - (1 - 0.99) = 0.94 = \beta$

$F(1.632) \sim 0.95$

$F(2.374) \sim 0.99$

from t-table when $n-1 = 60$

power $= 1 - \beta = 1 - 0.94 = 0.06$

② $\hat{p} = \frac{36}{45} = 0.80$

ⓐ $\alpha = 0.10$

$H_0: p \geq 0.70$

$H_1: p < 0.70$

$\frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} \longrightarrow$ normal distribution

one sided test,

$\boxed{\text{reject } H_0 \text{ if } \hat{p} < p_0 - z_\alpha \sqrt{p_0(1-p_0)/n}}$

$\hat{p} < 0.7 - 1.28\sqrt{0.7 \times 0.3/45}$

$\hat{p} < 0.612$

Since $\hat{p} = 0.80 > 0.612$, we do not reject H_0.

② (b) p-value,

$$\frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.80-0.70}{\sqrt{0.7\times 0.3/45}} = 1.464$$

$$P(Z<1.464) \longrightarrow F(1.464)$$

$$\left.\begin{array}{l} P(Z<1.46) = 0.9279 \\ P(Z<1.47) = 0.9292 \end{array}\right\} P(Z<1.464) \sim 0.9285$$

$$p\text{-value} \sim 0.9285 \geq \alpha = 0.1$$

So, we do not reject H_0.

② (c) Type II error probability and power?

$p = 60$

$$\beta = P(\text{Do not Reject } H_0 \mid p = 0.60)$$

$$= P(\hat{p} \geq 0.613 \mid p = 0.60)$$

$$= P\left(\frac{\hat{p}-0.60}{\sqrt{0.6\times 0.4/45}} \geq \frac{0.613-0.60}{\sqrt{0.6\times 0.4/45}}\right)$$

$$= P(Z \geq 0.178) = 1 - F(0.178)$$

$$\sim 1 - 0.57 = 0.43 \sim \beta$$

$$\text{power} = 1-\beta = 1-0.43 = 0.57$$

③ $n=10$, $s=22$

ⓐ $\alpha = 0.05$

$H_0: \sigma \leq 15$

$H_1: \sigma > 15$

$$\boxed{\frac{s^2(n-1)}{\sigma^2} \longrightarrow \chi^2 \text{ distribution with } n-1 \text{ degrees of freedom}}$$

one sided test,

$$\boxed{\text{reject } H_0 \text{ if } \frac{s^2(n-1)}{\sigma_0^2} > \chi^2_{n-1,\alpha}}$$

$$\frac{s^2(n-1)}{\sigma_0^2} > \chi^2_{9,\,0.05}$$

$$= \frac{s^2(9)}{(15)^2} > 16.92$$

$$= s^2 > \frac{16.92\,(15)^2}{9} = 423$$

Since $s = 22$ and $s^2 = 484 > 423$

Therefore, we reject H_0.

ⓑ p-value,

$$\frac{s^2(n-1)}{\sigma_0^2} = \frac{484\,(9)}{225} = 19.36$$

$P(\chi^2_9 > 19.36) \sim 0.025$

one sided test,

p-value $\sim 0.025 < 0.05$

So, we reject H_0.

④ $\alpha = 0.10$, $n_1 = 16$, $n_2 = 20$

$s_1 = 20$, $s_2 = 16$

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$$\boxed{\text{reject } H_0 \text{ if } F = \frac{s_1^2}{s_2^2} > F_{n_1-1,\, n_2-1,\, \alpha/2}}$$

$$F = \frac{s_1^2}{s_2^2} = \frac{20^2}{16^2} > F_{15,19,0.05}$$

$$\downarrow \qquad \downarrow$$

$$1.562 < 2.23$$

So, we do not reject H_0.