ECON 253 – HW3

① Poisson process $\Rightarrow P(x) = \dfrac{e^{-\lambda} \cdot \lambda^x}{x!}$

x = # of occurrences desired

$\lambda = 2.(1,5) = 3$ $\searrow$ $E[x] = \lambda . t = 2.(1,5) = 3$

λ = mean number of occurrences in a given interval (expected number)

$\underbrace{P(X \geq 1)}_{\text{at least one time, the phone rings}} = ?$

$P(X \geq 1) = 1 - \underbrace{P(X=0)}_{\text{the phone does not ring.}}$

$$= 1 - \frac{e^{-3} . (+3)^0}{0!} = 1 - e^{-3} = 1 - 0.05 = 0.95$$

probability $\Rightarrow$ 95 %

② $\mu = 1$, $\sigma = 0.10$, $n = 25$

$\boxed{z = \dfrac{x - \mu}{\sigma}}$

ⓐ $P(0.80 < x < 0.85)$

$$= P\left(\frac{0.80 - \mu}{\sigma} < \frac{x - \mu}{\sigma} < \frac{0.85 - \mu}{\sigma}\right)$$

$$= P\left(\frac{0.80 - 1}{0.1} < z < \frac{0.85 - 1}{0.1}\right)$$

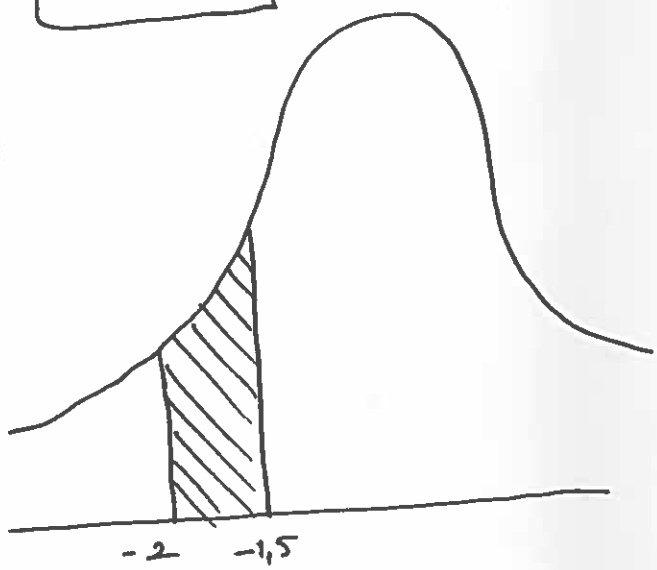
$$= P(-2 < z < -1.5)$$

$$= (1 - P(z < 1.5)) - (1 - P(z < 2))$$

$$= 1 - 0.933 - (1 - 0.977)$$

$$= 0.067 - 0.023 = 0.044$$

4.5 %



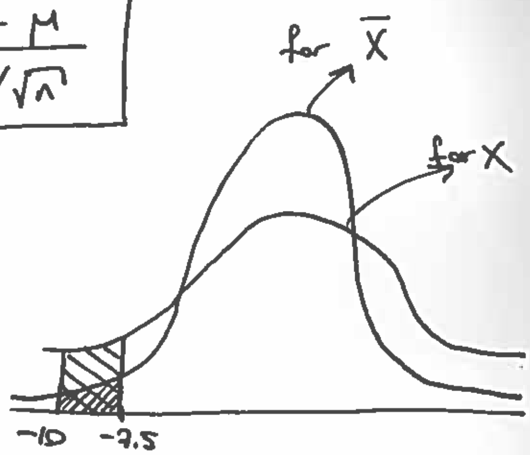
(2) (b) $P(0.80 < \bar{X} < 0.85)$

$$\boxed{z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}$$

$$= P\left(\frac{0.80-1}{0.1/\sqrt{25}} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < \frac{0.85-1}{0.1/\sqrt{25}}\right)$$

$$= P(-10 < z < -7.5)$$

It is very close to 0.



The area below $\bar{X}$ between 0.80 and 0.85 is far smaller than the area below X in this range.

(3) uniform distribution

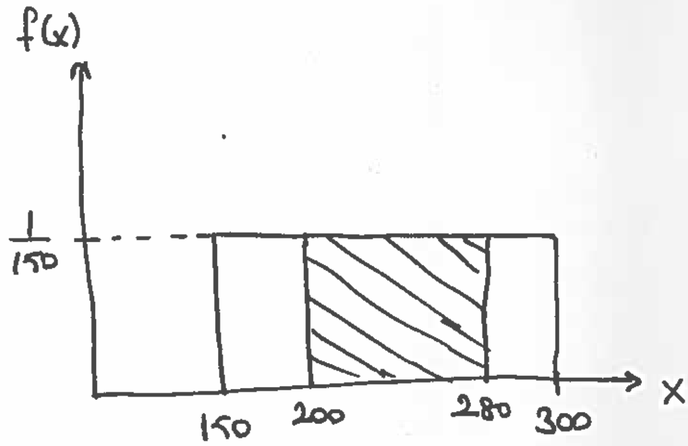
$$f(x) = \frac{1}{b-a}$$

$$= \frac{1}{300-150} = \frac{1}{150}$$

$$P(200 \leq x \leq 280) = \frac{1}{150} \cdot 80$$

$$= \frac{80}{150} = 0.54$$

54 %



another solution;

$$P(200 \leq x \leq 280) = \int_{200}^{280} f(x).dx = \int_{200}^{280} \frac{1}{150}\, dx = \left.\frac{x}{150}\right|_{200}^{280}$$

$$= \frac{280-200}{150} = \frac{80}{150}$$

$$= 0.54$$

54 %

④ exponential probability distribution

$\mu = 3$, $P(X \leq 2) = ?$

$F(t) = 1 - e^{-\lambda t}$

If mean time between arrivals is 3 minutes, about 1/3 cars arrive in a minute. Then, $\lambda = \frac{1}{3}$

$$\lambda t = \frac{1}{3} \cdot 2 = \frac{2}{3}$$

$$P(X \leq 2) = 1 - e^{-\lambda t}$$

$$= 1 - e^{-\frac{1}{3} \cdot 2} = 1 - e^{-2/3} = 1 - 0.513 = 0.487$$

48%

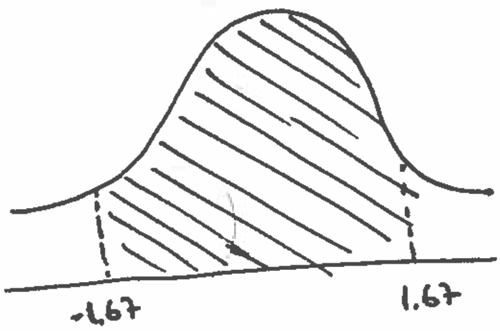
⑤ sampling distribution of the sample proportion

$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$ $p = 0.2$, $n = 270$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.2 \times 0.8}{270}} = 0.024$$

$$P(0.16 < \hat{p} < 0.24) = P\left(\frac{0.16 - p}{\sigma_{\hat{p}}} < \frac{\hat{p} - p}{\sigma_{\hat{p}}} < \frac{0.24 - p}{\sigma_{\hat{p}}}\right)$$

$$= P\left(\frac{0.16 - 0.2}{0.024} < Z < \frac{0.24 - 0.2}{0.024}\right)$$



$$= P(-1.67 < Z < 1.67)$$

$$= P(Z < 1.67) - (1 - P(Z < 1.67))$$

$$= 0.9525 - (1 - 0.9525)$$

$$= 0.9525 - 0.0475$$

$$= 0.905$$

90%

⑥ $n=30$, $\mu=185.1$, $\alpha=0.05 \longrightarrow 95\%$ confidence interval

ⓐ CI for μ (σ^2 known) $\longrightarrow \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\qquad \sigma^2=7$, $\sigma=\sqrt{7}$

$$185.1 \pm z_{0.025} \sqrt{\frac{7}{30}}$$

$$= 185.1 \pm 1.96 \times 0.48$$

$$= 185.1 \pm 0.9408$$

$\Rightarrow 184,16 < \mu < 186,04 \longrightarrow CI_{95\%} = (184.16, 186.04)$

ⓑ $s^2=7$ $\qquad$ CI for μ (σ^2 unknown) $\longrightarrow \bar{X} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}$

$s=\sqrt{7}$

$$185.1 \pm t_{29,0.025} \sqrt{\frac{7}{30}}$$

$$= 185.1 \pm 2.045 \times 0.48$$

$$= 185.1 \pm 2.613$$

$\Rightarrow 182.487 < \mu < 187.713 \longrightarrow CI_{95\%} = (182.487, 187.713)$

ⓒ Since $n=60$ is large, we can use both t and z test.

t

$$185.1 \pm t_{59,0.025} \sqrt{\frac{7}{60}}$$

$$= 185.1 \pm 2(0.341)$$

$$= 185.1 \pm 0.682$$

$\Rightarrow 184.418 < \mu < 185.782$

$CI_{95\%} = (184.418, 185.782)$

z

$$185.1 \pm z_{0.025} \sqrt{\frac{7}{60}}$$

$$= 185.1 \pm 1.96(0.341)$$

$$= 185.1 \pm 0.668$$

$\Rightarrow 184.432 < \mu < 185.768$

$CI_{95\%} = (184.432, 185.768)$

Similar results