Discrete Probability Distributions

Exercises (Questions 4.11, 4.14, 4.18, 4.22, 4.23, 4.28, 4.37, 4.38, 4.46, 4.58, 4.64, 4.67, 4.79, 4.82 in the 7th edition of the textbook)

4.11 Show the probability distribution function of the number of heads when three fair coins are tossed independently.

X: # of H when 3 coins are tossed.
S has 8 elements. (2^3)
HTT, THT, TTH

$P(X=0) = 1/8$, $P(X=1) = 3/8$
$P(X=2) = 3/8$, $P(X=3) = 1/8$

4.14 American Travel Air has asked you to study flight delays during the week before Christmas at Midway Airport. The random variable X is the number of flights delayed per hour.

X	0	1	2	3	4	5	6	7	8	9
P(x)	0.10	0.08	0.07	0.15	0.12	0.08	0.10	0.12	0.08	0.10
F(X)	0.10	0.18	0.25	0.4	0.52	0.6	0.7	0.82	0.9	1

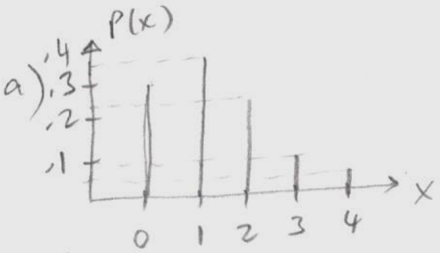
a. What is the cumulative probability distribution?
b. What is the probability of five or more delayed flights? $1-F(4) = 1-0.52 = 0.48$
c. What is the probability of three through seven (inclusive) delayed flights?

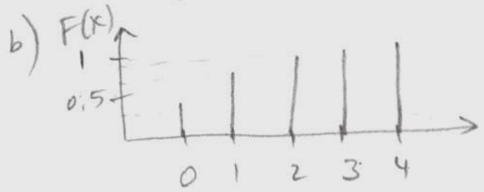
$\sum_{i=3}^{7} P(X=i) = 0.15 + 0.12 + 0.08 + 0.10 + 0.12 = 0.57$

4.18 An automobile dealer calculates the proportion of new cars sold that have been returned a various numbers of times for the correction of defects during the warranty period. The results are shown in the following table.

Number of returns	0	1	2	3	4
Proportion P(X)	0.28	0.36	0.23	0.09	0.04
F(X)	0.28	0.64	0.87	0.96	1

a. Draw the probability distribution function.
b. Calculate and draw the cumulative probability function.
c. Find the mean of the number of returns of an automobile for corrections for defects during the warranty period.
d. Find the variance of the number of returns of an automobile for corrections for defects during the warranty period.

a)


b)


c) $\mu = 0 + 0.36 + 0.46 + 0.27 + 0.16$
$\mu = 1.25$

d) $\sigma^2 = \sum x^2 P(x) - \mu^2$
$= 0 + 0.36 + 0.92 + 0.81 + 0.64 - (1.25)^2$

125
125
625
250
125
1.5625

36
92
81
64
2.73
1.5625
1.1675

0.23 × 4 = 0.92 .16 × 4 = 64

$\sigma^2 = 2.73 - 1.5625 = 1.1675$

$\frac{1-(0.6)^2}{1-0.6} = \frac{1-0.36}{0.4} = \frac{0.64}{0.4}$

$1+0.6+0.36 = 1.96$

4.22 a. A very large shipment of parts contains 10% defectives. Two parts are chosen at random from the shipment and checked. Let the random variable X denote the number of defectives found. Find the probability function of this random variable.

b. A shipment of 20 parts contains two defectives. Two parts are chosen at random from the shipment and checked. Let the random variable Y denote the number of defectives found. Find the probability function of this random variable. Explain why your answer is different from that for part (a).

c. Find the mean and variance of the random variable X in part (a).

d. Find the mean and variance of the random variable Y in part (b).

$\frac{\binom{2}{0}\binom{18}{2}}{\binom{20}{2}} =$

(a) $P(X=0) = (0.9)^2 = 0.81$

$P(X=1) = (0.9)(0.1) \cdot 2 = 0.18$

$P(X=2) = (0.1)(0.1) = 0.01$

(b) $P(X=0) = \frac{18}{20} \cdot \frac{17}{19} = \frac{153}{190}$

$P(X=1) = \frac{18}{20} \cdot \frac{2}{19} + \frac{2}{20} \cdot \frac{18}{19} = \frac{36}{190}$

$P(x=2) = \frac{2}{20} \cdot \frac{1}{19} = \frac{1}{190}$

B/c in part (b) prob. is dependent on the 1st draw.

(c) $\mu_{xa} = 0 + 0.18 + 0.02 = 0.20$

$\sigma_X^2 = \sum x^2 P(x) - \mu^2 = 0.18 + 0.04 - 0.04 = 0.18$

(d) $\mu_{xb} = 36/190 + 2/190 = 38/190 = 0.2$

$\sigma_{xa}^2 = 36/190 + 4/190 - 0.04 = \frac{4}{19} - \frac{4}{100} = 0.1705$

4.23 A student needs to know details of a class assignment that is due the next day and decides to call fellow class members for this information. She believes that for any particular call the probability of obtaining the necessary information is 0.40. She decides to continue calling class members until the information is obtained. Let the random variable X denote the number of calls needed to obtain the information.

a. Find the probability function of X.

b. Find the cumulative probability function of X.

c. Find the probability that at least three calls are required.

$P(1) = 0.4 \quad P(2) = (0.6)(0.4)$

$P(3) = (0.6)^2(0.4)$

$P(X=x) = (0.6)^{x-1}(0.4)$

(b) $F(1) = 0.4$, $F(2) = 0.4[1+0.6]$

$F(3) = 0.4[1+0.6+0.6^2]$

$F(X=x) = 0.4\left[\frac{1-0.6^x}{1-0.6}\right] = 1-0.6^x$

(c) $1-F(2) = 1-0.64 = 0.36$

4.28 A factory manager is considering whether to replace a temperamental machine. A review of past records indicates the following probability distribution for the number of breakdowns of this machine in a week.

Number of breakdowns	0	1	2	3	4
Probability	0.10	0.26	0.42	0.16	0.06

a. Find the mean and standard deviation of the number of weekly breakdowns.

b. It is estimated that each breakdown costs the company \$1,500 in lost output. Find the mean and standard deviation of the weekly cost to the company from breakdowns of this machine.

(a) $\mu_X = 0.26 + 0.84 + 0.48 + 0.24 = 1.82$

$\sigma_X^2 = \sum x^2 P(x) - \mu^2 = 1.0276$

$\sigma_X = 1.0137$

(b) $Y = 1500X$

$E(Y) = 1500\mu_X = \$2730$

$\sigma^2(Y) = (1500)^2 \sigma_X^2$

$\sigma(Y) = 1500\sigma_X = \$1520.5$

4.37 A public interest group hires students to solicit donations by telephone. After a brief training period students make calls to potential donors and are paid on a commission basis. Experience indicates that early on these students tend to have only modest success and that 70% of them give up their jobs in their first 2 weeks of employment. The group hires six students, which can be viewed as a random sample.

a. What is the probability that at least two of the six will give up in the first 2 weeks?
b. What is the probability that at least two of the six will not give up in the first 2 weeks?

(a) $P(X \geq 2) = 1 - P(X < 2)$
$= 1 - (P(0) + P(1))$
$= 1 - [C_0^6 (0.7)^0 (0.3)^6 + C_1^6 (0.7)^1 (0.3)^5]$
$= 1 - (0.3)^6 - 6 \cdot (0.7)(0.3)^5$
$= 0.9891$

(b) $P(X \leq 4) = 1 - P(5) - P(6)$
$= 1 - C_5^6 (0.7)^5 (0.3)^1 - C_6^6 (0.7)^6$
$= 0.5798$

4.38 Suppose that the probability is 0.2 that the value of the U.S. dollar will rise against the Chinese yuan over any given week and that the outcome in one week is independent of that in any other week. What is the probability that the value of the U.S. dollar will rise against the Chinese yuan at least twice over a period of 7 weeks?

$P(X \geq 2) = 1 - P(X < 2)$
$= 1 - P(0) - P(1)$
$= 1 - C_0^7 (0.2)^0 (0.8)^7 - C_1^7 (0.2)^1 (0.8)^6$
$= 0.423$

4.46 A campus finance officer finds that, for all parking tickets issued, fines are paid for 78% of the tickets. The fine is $2. In the most recent week 620 parking tickets have been issued.

a. Find the mean and standard deviation of the number of these tickets for which the fines will be paid.
b. Find the mean and standard deviation of the amount of money that will be obtained from the payment of these fines. $Y = 2X$

(a) $E(X) = np = 620(0.78)$
$= 483.6$
$\sigma_X^2 = np(1-p)$
$\sigma_X = \sqrt{620(0.78)(0.22)}$
$= 10.3146$

(b) $E(Y) = 2E(X) = 967.2$
$\sigma_Y = 2\sigma_X = 20.6292$

4.58 A bank executive is presented with loan applications from 10 people. The profiles of the applicants are similar, except that 5 are minorities and 5 are not minorities. In the end the executive approves 6 of the applications. If these 6 approvals are chosen at random from the 10 applications, what is the probability that less than half the approvals will be of applications involving minorities?

$P(X = \cancel{0}, 1, \text{or } 2)$
$= \frac{C_1^5 C_5^5}{C_6^{10}} + \frac{C_2^5 C_4^5}{C_6^{10}}$

$$= \frac{\frac{5!}{4!} \cdot 1 + \frac{5!}{3!2!} \frac{5!}{4!}}{\frac{10!}{6!4!}} = \frac{5 + \frac{5 \cdot \cancel{4}^{2}}{\cancel{2}} \cdot 5}{\frac{10 \cdot \cancel{9}^{3} \cdot \cancel{8} \cdot 7}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2}}} = \frac{55}{210} = 0.2619 = \frac{11}{42}$$

$$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

4.64 Customers arrive at a busy checkout counter at an average rate of three per minute. If the distribution of arrivals is Poisson, find the probability that in any given minute there will be two or fewer arrivals.

$\lambda = 3/\text{minute}$

$P(X \le 2) = ?$

$P(X=0) + P(X=1) + P(X=2)$

$= \frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!} = 0.4232$

4.67 Records indicate that, on average, 3.2 breakdowns per day occur on an urban highway during the morning rush hour. Assume that the distribution is Poisson.

a. Find the probability that on any given day there will be fewer than two breakdowns on this highway during the morning rush hour.

b. Find the probability that on any given day there will be more than four breakdowns on this highway during the morning rush hour.

$\lambda = 3.2/\text{day}$

a) $P(X<2) = 0.1712$

b) $P(X>4) = 1 - P(X \le 4)$

$= 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$= 0.2194$

Quiz **4.79** Consider the joint probability distribution:

		X		
		1	2	
Y	0	0.30	0.20	0.50
	1	0.25	0.25	0.50
		0.55	0.45	

a. Compute the marginal probability distributions for X and Y.

b. Compute the covariance and correlation for X and Y.

c. Compute the mean and variance for the linear function $W = 2X + Y$.

$\mu_X = 1(0.55) + 2(0.45)$
$= 1.45$

$\mu_Y = 0.50$

$\sigma_X^2 = E(X^2) - \mu_X^2$
$= 1(0.55) + 4(0.45) - (1.45)^2$
$= 2.35 \Rightarrow \sigma_X = 1.5329$

$\sigma_Y^2 = 0.25 \Rightarrow \sigma_Y = 0.5$

b) $Cov(X,Y) = E(XY) - \mu_X\mu_Y$

$= [0(0.30) + 0(0.20) + 1(0.25) + 2(0.25)] - (1.45)(0.50)$

$= 0.25 + 0.50 - 0.725 = 0.025$

$\rho = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{0.025}{(1.5329)(0.5)} = 0.0326$

c) $E(W) = 2E(X) + E(Y) = 2(1.45) + (0.50) = 3.40$

$\sigma_W^2 = 4\sigma_X^2 + \sigma_Y^2 + 2 \cdot 2 Cov(X,Y)$

$= 4(2.35) + (0.25) + 4(0.025) = 9.75$

4.82 A researcher suspected that the number of between-meal snacks eaten by students in a day during final examinations might depend on the number of tests a student had to take on that day. The accompanying table shows joint probabilities, estimated from a survey.

Number of Snacks (Y)	Number of Tests (X) 0	1	2	3	
0	0.07	0.09	0.06	0.01	0.23
1	0.07	0.06	0.07	0.01	0.21
2	0.06	0.07	0.14	0.03	0.30
3	0.02	0.04	0.16	0.04	0.26
	0.22	0.26	0.43	0.09	

a. Find the probability function of X and, hence, the mean number of tests taken by students on that day.
b. Find the probability function of Y and, hence, the mean number of snacks eaten by students on that day.
c. Find and interpret the conditional probability function of Y, given that $X = 3$.
d. Find the covariance between X and Y.
e. Are number of snacks and number of tests independent of each other?

a) $\mu_X = 1.39$

b) $\mu_Y = 1.59$

c) $P(Y=0 \mid X=3) = \frac{0.01}{0.09} = 1/9$

$P(Y=1 \mid X=3) = 1/9$

$P(Y=2 \mid X=3) = 3/9$

$P(Y=3 \mid X=3) = 4/9$

d) $\mathrm{Cov}(X,Y) = E(XY) - \mu_X \mu_Y$

$= \underbrace{\cdots}_{9 \text{ terms}} - \mu_X \mu_Y$

$= 2.55 - (1.39)(1.59)$

$= 0.3399$

e) no, b/c $\mathrm{Cov}(X,Y) \neq 0$.