Example 5.1 Probability of Pipeline Failure (Cumulative Distribution Function)

A repair team is responsible for a stretch of oil pipeline 2 miles long. The distance (in miles) at which any fracture occurs can be represented by a uniformly distributed random variable, with probability density function

$$f(x) = 0.5$$

Find the cumulative distribution function and the probability that any given fracture occurs between 0.5 mile and 1.5 miles along this stretch of pipeline.

5.5 An analyst has available two forecasts, F_1 and F_2, of earnings per share of a corporation next year. He intends to form a compromise forecast as a weighted average of the two individual forecasts. In forming the compromise forecast, weight X will be given to the first forecast and weight $(1 - X)$ to the second, so that the compromise forecast is $XF_1 + (1 - X)F_2$. The analyst wants to choose a value between 0 and 1 for the weight X, but he is quite uncertain of what will be the best choice. Suppose that what eventually emerges as the best possible choice of the weight X can be viewed as a random variable uniformly distributed between 0 and 1, having the probability density function

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{for all other values of } x \end{cases}$$

a. Draw the probability density function.
b. Find and draw the cumulative distribution function.
c. Find the probability that the best choice of the weight X is less than 0.25.
d. Find the probability that the best choice of the weight X is more than 0.75.
e. Find the probability that the best choice of the weight X is between 0.2 and 0.8.

5.7 The incomes of all families in a particular suburb can be represented by a continuous random variable. It is known that the median income for all families in this suburb is \$60,000 and that 40% of all families in the suburb have incomes above \$72,000.

a. For a randomly chosen family, what is the probability that its income will be between \$60,000 and \$72,000?

b. Given no further information, what can be said about the probability that a randomly chosen family has an income below \$65,000?

a)
$P(60{,}000 < X < 72{,}000)$
$= P(X < 72000) - P(X < 60000)$
$= 0.6 - 0.5 = 0.1$

b) $P(X < 60000) < P(X < 65000) < P(X < 72000)$ and
$0.5 < P(X < 65000) < 0.6$.

5.18 Let the random variable Z follow a standard normal distribution.

a. The probability is 0.70 that Z is less than what number? $P(Z < Z_0) = 0.7 \Rightarrow Z_0 = 0.52$ (closest value)

b. The probability is 0.25 that Z is less than what number? $Z_0 = -0.67$

c. The probability is 0.2 that Z is greater than what number? $Z_0 = 0.84$

d. The probability is 0.6 that Z is greater than what number? $Z_0 = -0.25$

5.20 Let the random variable X follow a normal distribution with $\mu = 80$ and $\sigma^2 = 100$.

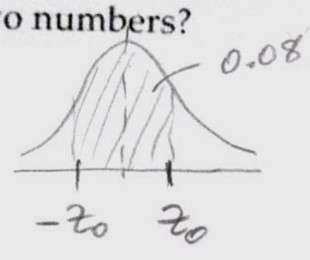
a. Find the probability that X is greater than 60. $P(X > 60) = P(Z > -2) = 0.9772$

b. Find the probability that X is greater than 72 and less than 82.
$P(72 < X < 82) = P(-0.8 < Z < 0.2) = 0.3674$

c. Find the probability that X is less than 55. $P(Z < -2.5) = 0.0062$

d. The probability is 0.1 that X is greater than what number?
$Z = 1.28$, $1.28 = \frac{X - 80}{10}$
$\Rightarrow X = 92.8$

e. The probability is 0.08 that X is in the symmetric interval about the mean between which two numbers?



$F(Z_0) = 0.54$
$\Rightarrow Z_0 = 0.10 \Rightarrow \frac{X - 80}{10} = \mp 0.10$
$\Rightarrow X = 79$ and $X = 81$

5.33 Tata Motors Ltd. purchases computer process chips from two suppliers and the company is concerned about the percentage of defective chips. A review of the records for each supplier indicates that the percentage defectives in consignments of chips follow normal distributions with the means and standard deviations given in the following table. The company is particularly anxious that the percentage of defectives in a consignment not exceed 5% and wants to purchase from the supplier that's more likely to meet that specification. Which supplier should be chosen?

	Mean	Standard Deviation
Supplier A	4.4	0.4
Supplier B	4.2	0.6

For A,

$P\left(Z < \frac{5-4.4}{0.4}\right) = 0.9332$

For B,

$P\left(Z < \frac{5-4.2}{0.6}\right) = 0.9082$

Choose A.

5.36 Scores on an achievement test are known to be normally distributed with a mean of 420 and a standard deviation of 80.

a. For a randomly chosen person taking this test, what is the probability of a score between 400 and 480?
b. What is the minimum test score needed in order to be in the top 10% of all people taking the test?
c. For a randomly chosen individual, state, without doing the calculations, in which of the following ranges his score is most likely to be: 400–439, 440–479, 480–519, or 520–559.
d. In which of the ranges listed in part (c) is the individual's score least likely to be?
e. Two people taking the test are chosen at random. What is the probability that at least one of them scores more than 500 points?

a) $F(0.75) - F(-0.25)$
$= 0.3721$

b) $P(Z > 1.28) = 0.10$
$\Rightarrow X = 522.4$

c) 400–439

d) 520–559

e) $P(X < 500) = P\left(Z < \frac{500-420}{80}\right)$
$= 0.8413$

$P(\text{at least one} > 500) = 1 - (0.8413)^2$
$= 0.2922$

5.52 Given an arrival process with $\lambda = 5.0$, what is the probability that an arrival occurs after $t = 7$ time units?

$P(T > 7) = 0.2019$

~~5.55 A professor sees students during regular office~~

$\text{Mean} = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{10}$

a) $P(X<20) = 1-e^{-20(1/10)} = 0.8647$

5.55 A professor sees students during regular office hours. Time spent with students follows an exponential distribution with a mean of 10 minutes.

a. Find the probability that a given student spends fewer than 20 minutes with the professor.
b. Find the probability that a given student spends more than 5 minutes with the professor.
c. Find the probability that a given student spends between 10 and 15 minutes with the professor.

b) $P(X>5) = 1-(1-e^{-5/10})$
$= 0.6065$

c) $P(10<X<15) = F(15)-F(10)$
$= 0.1447$

5.63 A random variable X is normally distributed with a mean of 100 and a variance of 100, and a random variable Y is normally distributed with a mean of 200 and a variance of 400. The random variables have a correlation coefficient equal to 0.5. Find the mean and variance of the random variable:

$$W = 5X - 4Y$$

$\mu_W = 5(100)-4(200) = -300$

$\sigma_W^2 = 5^2 \cdot 100 + 4^2 \cdot 400$
$- 2 \cdot 5 \cdot 4\,(0.5) \cdot 10 \cdot 20$

5.68 A consultant is beginning work on three projects. The expected profits from these projects are \$50,000, \$72,000, and \$40,000. The associated standard deviations are \$10,000, \$12,000, and \$9,000. Assuming independence of outcomes, find the mean and standard deviation of the consultant's total profit from these three projects.

$W = X_1 + X_2 + X_3$

$\mu_W = 50{,}000 + 72{,}000 + 40{,}000$

$\sigma_W^2 = (10{,}000^2 + 12000^2 + 9000^2)$

$\sigma_W = +\sqrt{\sigma_W^2}$