Statistics for Business and Economics

6th Edition

Chapter 9

Estimation: Additional Topics

Confidence Intervals for the Population Variance

Population Variance

- Goal: Form a confidence interval for the population variance, σ^2
- The confidence interval is based on the sample variance, s^2
- Assumed: the population is normally distributed

Confidence Intervals for the Population Variance

(continued)

Population Variance

The random variable

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$$

follows a chi-square distribution with (n – 1) degrees of freedom

The chi-square value $\chi^2_{n-1,\alpha}$ denotes the number for which

$$P(\chi^2_{n-1} > \chi^2_{n-1,\alpha}) = \alpha$$

Confidence Intervals for the Population Variance

(continued)

Population Variance

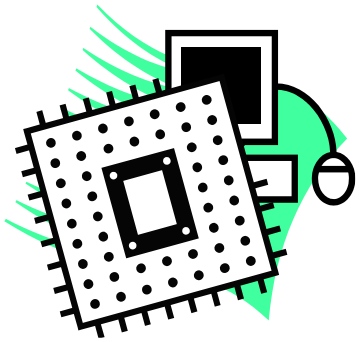
The (1 - α)% confidence interval for the population variance is

$$\frac{(n-1)s^2}{\chi^2_{n-1,\,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,\,1-\alpha/2}}$$

Example

You are testing the speed of a computer processor.
You collect the following data (in Mhz):

	CPU_x
Sample size	**17**
Sample mean	**3004**
Sample std dev	**74**



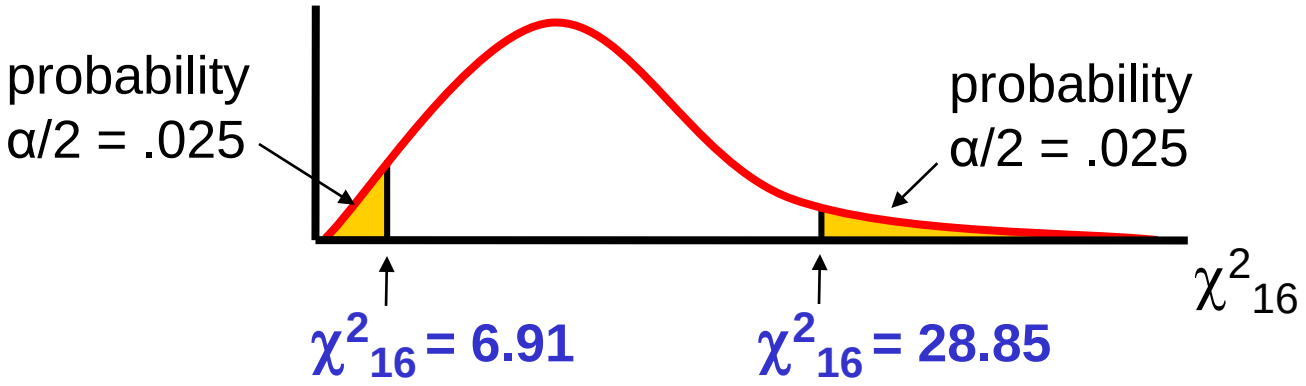
Assume the population is normal.
Determine the 95% confidence interval for σ_x^2

Finding the Chi-square Values

- n = 17 so the chi-square distribution has (n – 1) = 16 degrees of freedom
- α = 0.05, so use the the chi-square values with area 0.025 in each tail:

$$\chi^2_{n-1,\alpha/2} = \chi^2_{16,0.025} = 28.85$$

$$\chi^2_{n-1,1-\alpha/2} = \chi^2_{16,0.975} = 6.91$$



Calculating the Confidence Limits

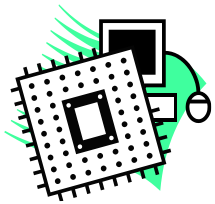
- The 95% confidence interval is

$$\frac{(n-1)s^2}{\chi^2_{n-1,\,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,\,1-\alpha/2}}$$

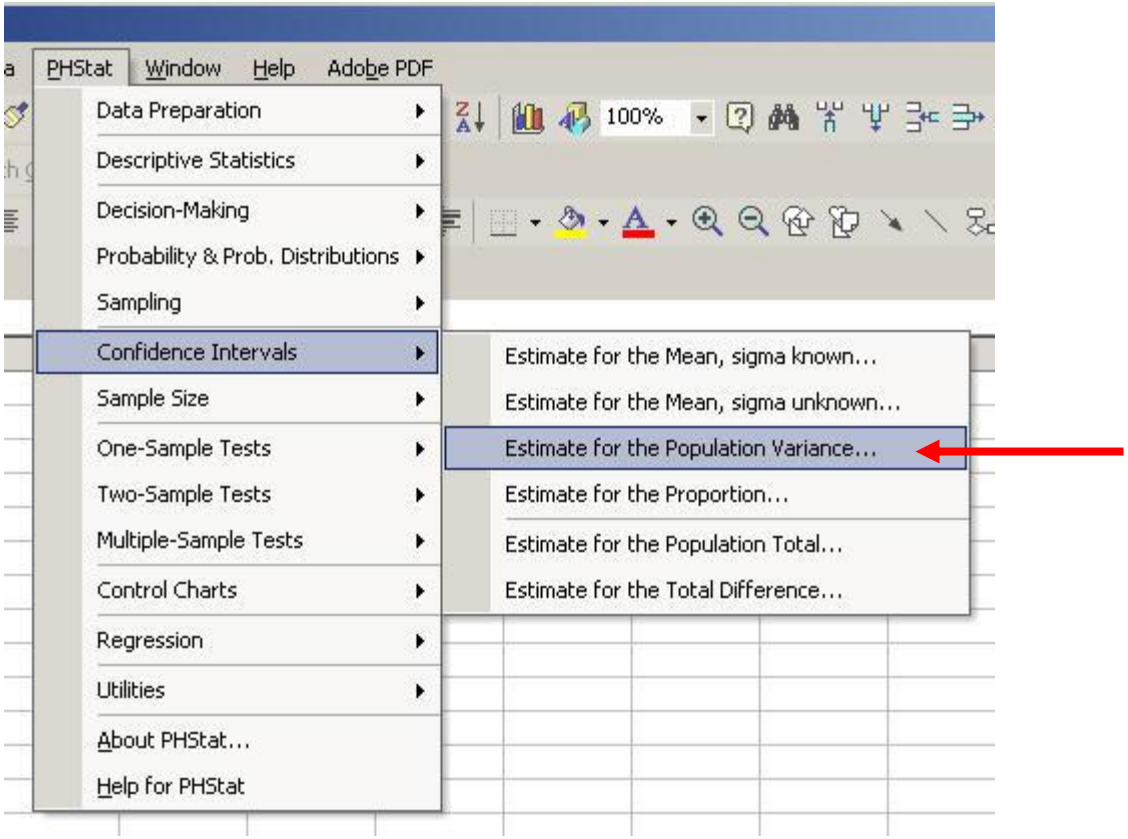
$$\frac{(17-1)(74)^2}{28.85} < \sigma^2 < \frac{(17-1)(74)^2}{6.91}$$

$$3037 < \sigma^2 < 12683$$

Converting to standard deviation, we are 95% confident that the population standard deviation of CPU speed is between 55.1 and 112.6 Mhz

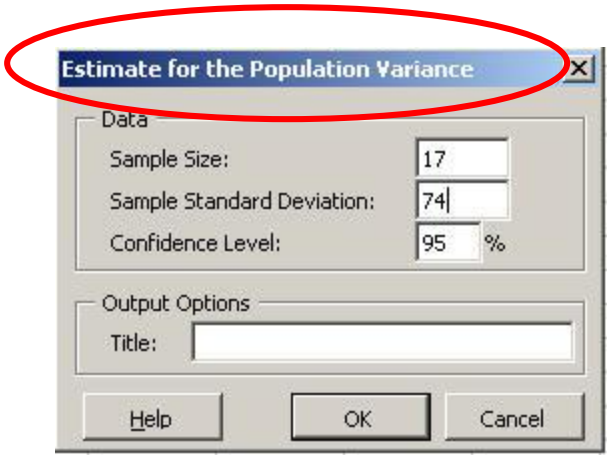


Sample PHStat Output



Sample PHStat Output

(continued)



	A	B	C	D	E
1	**Confidence Interval Estimate for the Population Variance**				
2					
3	**Data**				
4	**Sample Size**	**17**			
5	**Sample Standard Deviation**	**74**			
6	**Confidence Level**	**95%**			
7					
8	Intermediate Calculations				
9	Degrees of Freedom	16			
10	Sum of Squares	87616			
11	Single Tail Area	0.025			
12	Lower Chi-Square Value	6.907664			
13	Upper Chi-Square Value	28.84532			
14					
15	**Results**				
16	**Interval Lower Limit for Variance**	**3037.442**			
17	**Interval Upper Limit for Variance**	**12683.88**			
18					
19	**Interval Lower Limit for Standard Deviation**	**55.11299**			
20	**Interval Upper Limit for Standard Deviation**	**112.6227**			
21					
22	*Assumption:*				
23	**Population from which sample was drawn has an approximate normal distribution.**				
24					

Input

Output